## Inverse square law

Passion for science

| Number | 133710-EN | Topic | Light, energy of electromagnetic radiation |  |  |
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## Objective

Investigating how the light intensity depends on the distance of the light source.
For light that is not absorbed in air, the intensity is expected to decrease like the square of the distance.

## Principle

An incandescent lamp is powered by a stabilised power supply in order to keep its emitted power constant. The intensity of the light (including infrared radiation) is measured using a broadband detector.

## Equipment

(Detailed equipment list on p. 4)
277022 Stefan-Boltzmann's lamp
287181 IR-sensor broadband
251560 Battery box
Power supply
Multimeter
Ruler
Lab leads
Stand material

## Distances in the universe

If you know the luminosity of an object and measure the intensity of the light that reaches us, the inverse square law can be used to calculate the distance to the object. This is an indispensable method in astronomy.


## Procedure

## Preparations

A dimly lit room is fine for the measurements, but there is no need for complete darkness. What is important is to avoid large changes in illumination.
The glass at the "top" of the light bulb (opposite the socket) is often thickened into a kind of lens. To avoid this, use the light exiting through the side of the bulb. Check that the filament is vertical.

Connect a power supply to the two lower sockets on the lamp.
Connect the broadband sensor to the Sensor input on the battery box. Orient the plug carefully - the socket has more holes than there are pins in the plu.
Place the lamp on e.g. a small wooden block. The lamp base may be fixed with a little adhesive tape.
The sensor must be at the same height as the centre of the light bulb. The distance between lamp and sensor must be able to be varied.

## Measuring distances

You can assume that there is approx. 13 mm from the front of the sensor to the photo-sensitive surface. (This distance cannot be measured.)
It is the distance $r$ between the filament and the sensitive surface in the sensor that enters the inverse square low, but for practical reasons we measure a different distance $L$ and correct the numbers afterwards by a fixed displacement that we call $r_{0}$ :

$$
r=L-r_{0}
$$

For example, you can place a ruler with its zero against the base of the lamp, and place the sensor in a stand base on top of the ruler. Keep the ruler in place while sliding the sensor base, and $L$ will be easy to read off the ruler precisely:


Measure the distances necessary to determine $r_{0}(=L-r)$ reasonably precisely. If you turn up the power supply to approx. 2 V to make the filament glow faintly, it is easier to see its position. It is not necessarily in the centre of the bulb!
(Write down a calculation example giving $r$ from L.)

## Measurements

Adjust the voltage across the lamp to 12.0 V .
The output voltage from the sensor is measured with a multimeter, The voltage is proportional with the intensity of the light falling on the sensor element. A calibration factor "CAL" is specified on a label on the sensor. (Write down this value now!) Multiply the voltage measured in $V$ with this factor to obtain the intensity in $\mathrm{W} / \mathrm{m}^{2}$.
Wait until the lamp has been lit for approx. 5 minutes before the measurements start.
Now, the intensity must be measured for different distances. A suitable distribution of distances $r$ could be this:

$$
10,11,12,13,14,15,17,19,22,26,30,50 \mathrm{~cm}
$$

(Keep the largest distance below 60-70 cm; the uncertainties get large when the light gets too dim)
This series of measurements must of course be done carefully, but it should not take too long in order to avoid changes in the measurement conditions.
Read and write down the output of the sensor - wait until later to convert the voltages to intensities. Vi will refer to these measurements as Series 1.
You can use a table like this:


Finally, make a supplementary Series 2 with three measurements closer to the filament. Start by moving the detector so close that the output voltage is just below 4 V . Choose the other two distances further away, but closer than 10 cm (distributed evenly).
These data are not used in the primary analysis - see details in Calculations.

## Theory

We assume the filament is so small that it can be treated as a point.

Normal clean and dry air absorbs virtually no light at all. The light energy emitted from the filament will therefore not disappear as the distance increases but the energy will spread over a larger and larger area. The radiation intensity $I_{R}$ (power per area) will therefore decrease. If the light is distributed evenly in all directions, the total power $P_{\mathrm{R}}$ must at a distance of $r$ be distributed over an area given by $A=4 \pi r^{2}$ (the surface of a sphere). The radiation intensity is then

$$
I_{\mathrm{R}}=\frac{P_{\mathrm{R}}}{4 \cdot \pi \cdot r^{2}}=I_{0} \cdot \frac{1}{r^{2}}
$$

where $I_{0}$ is a constant.
We observe that the intensity is inversely proportional to the square of the distance.
When measuring the intensity, you will often encounter a small offset from zero. This means that a graph of $I s$ as a function of $1 / r^{2}$ doesn't necessarily pass through $(0,0)$.

## Calculations

Use a spreadsheet and place the offset ro in a separate cell. (If necessary, this can be adjusted for optimum fit with the data points.)

Fill in the calculated columns with $r, 1 / r^{2}$ and $I_{s}$ (all in SI units):

| $C A L:$ | $\left(\mathrm{W} / \mathrm{m}^{2}\right) / \mathrm{V}$ |
| ---: | :--- |
| $r_{0}:$ | cm |


| Measured |  | Calculated |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{L}$ | $\boldsymbol{U}$ | $\boldsymbol{r}$ | $\mathbf{1} / \mathrm{r}^{2}$ | $\boldsymbol{I}_{\mathbf{s}}$ |
| cm | V | m | $\mathrm{m}^{-2}$ | $\mathrm{~W} / \mathrm{m}^{2}$ |
|  |  |  |  |  |

Plot the data from Series 1 in a coordinate system with $1 / r^{2}$ horizontally and the corresponding values of $I_{s}$ horizontally. The data points should fall on a straight line. (Let the spreadsheet draw the best straight line.)
In case the data points curves systematically, you can adjust $r_{0}$ to try to straighten out the curve - but the value must still be reasonable in view of the setup.
Finally, copy the graph (including data and trend line from series 1), and add the data points from Series 2 as a separate data series. Extend the trend line from series 1.


The same amount of radiation energy passes areas that grow as distance squared

## Discussion and evaluation

## Series 1

Describe the consistency between your measurements and the expected behaviour.

## Series 2

Do these measurements fall on the same straight line as Series 1? Try to explain a possible deviation.

## Supplementary problem

Select one of the measurements with $r \approx 120 \mathrm{~mm}$.
Calculate the area of a sphere with radius = the value chosen for $r$.
Calculate - based on the corresponding intensity - the total power radiated by the filament.
The nominal electric power of the lamp bulb is 21 W . Compare with the results above.

## Teacher's notes

## Concepts used

Power
Radiation intensity
Mathematical skills
Area of a sphere
Using spreadsheet
Linear fit
About the equipment
A very good match can be expected with the inverse square law when using the light sent out through the side of the bulb, as described.
It is, however, a prerequisite that the filament is vertical. If it isn't, the lamp can be rotated in the holder after loosening the grub screw holding the socket. (Use a 2 mm hex wrench.)
Careful! Don't crush the glass.

## Detailed equipment list

Specifically for the experiment
277022 Stefan-Boltzmanns lamp
287281 IR-sensor broadband
251560 Battery box

## Standard lab equipment

361600 Power supply (or similar)
386135 Multimeter (or similar)
105720 Safety cable 50 cm black (Qty. 2)
105721 Safety cable, 50 cm , red (Qty. 2)
140510 Ruler, 100 cm

