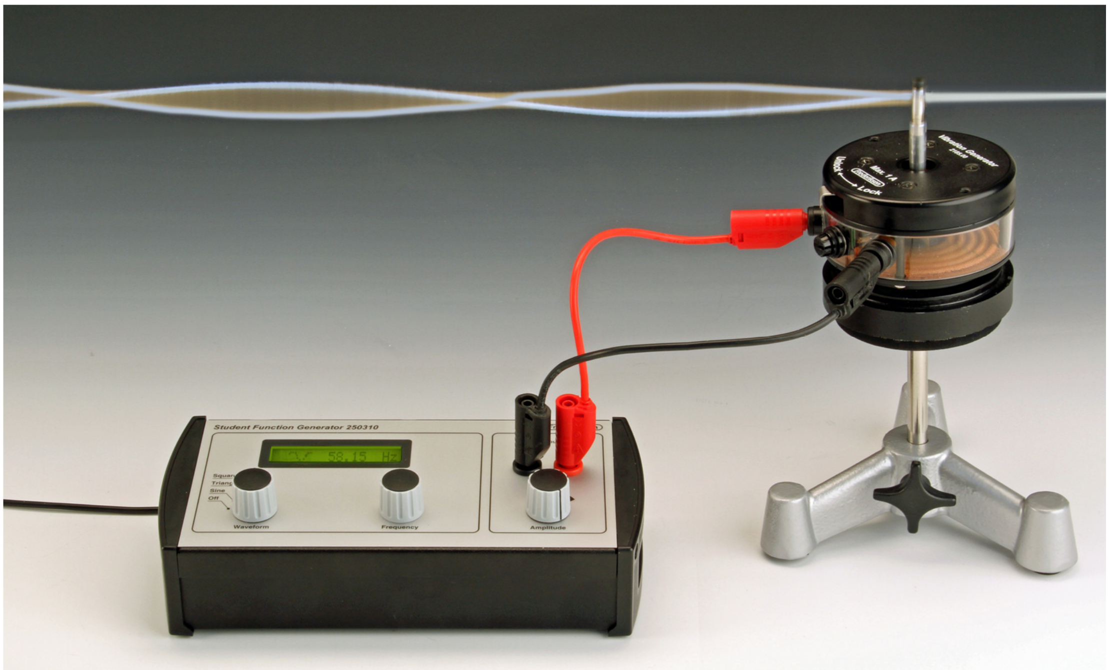
Resonance – standing waves on a string

Experiment number	132860-EN	*Subject*	Waves				
Version	2017-07-13 / HS	*Type*	Student exercise	*Suggested for*	grade 10-12	*p.*	1/4



Objective

A study of standing waves on a tight string, with a focus on the frequency of the harmonics.

The relationship between the frequency of the natural oscillations and the thickness and tension of the string is investigated.

The propagation velocity of the waves is determined.

Principle

A function generator is connected to a vibrator to emit a wave train along the string. The wave train is reflected at both ends of the string. At certain frequencies, strong standing waves are observed. The phenomenon is called resonance. At the resonance frequencies it is easy to determine the wavelength.

Equipment

(Detailed list of equipment on p. 4)

Function generator (with frequency display)
Vibrator
String, cord, fishing line or similar
Stand material, including a pulley
Dynamometer (spring balance)
Lab leads / safety cables
Tape measure or ruler

The principle of string instruments

Stringed instruments – guitar, double bass, banjo, violin, concert grand piano etc. – all follow the same basic principles for tone generation.

The pitch is determined primarily by the length of the string, its mass per unit length, and its tension.

Photo source: See p. 4

Frederiksen Scientific A/S
Viaduktvej 35 · DK-6870 Ølgod
Tel. +45 7524 4966
Fax +45 7524 6282
info@frederiksen.eu
www.frederiksen.eu

Standing waves and resonance

The *once* reflected wave together with the original wave will create a pattern of so-called standing waves that only oscillates up and down.

If the *twice* reflected wave moves in step with the original wave, they will reinforce each other. This is called resonance and only happens at certain frequencies where the amplitude of the standing wave becomes very large. See photo on p. 1 and drawing below.

A place with large oscillations is called an *anti-node*.

Points that hardly moves are called *nodes*.

The distance between neighbouring nodes is one half of the wavelength.

At the *fundamental* frequency of the string, only one anti-node occurs. Resonances with more anti-nodes are called *harmonics*.

On the drawing below, there are 3 anti-nodes and 4 nodes (including the end points).

Setting up

While setting up, lock the vibrator piston – and do the same when you're cleaning up. During measurements, put the lever in the *Unlock* position.

A sturdy setup is achieved with a couple of table clamps to fix the vertical rods. The clamps are best placed on adjacent edges around a table corner.

The vibrator must be placed close to one end of the string. Run the string once around the screw at the top of the string holder. There should be no net force on the vibrator from the string.
In the opposite end of the string, it goes via a pulley to a newton meter (dynamometer)

The pulley can be considered a fixed point relative to the vibrations of the string. Surprisingly, the same is true for the vibrator – it is easily seen that the movement is minimal here at a resonance.

The part of the string that moves freely lies between the pulley and the vibrator. Its length is called L .

The string tension is adjusted by moving the top of the dynamometer up and down.
Adjust for a tension of 5 N.

Connect the function generator to the vibrator. Set the waveform switch to *Sine*.

Measurements

Ask your teacher which of the following sections you should complete.

Measurements – 1

Start with a low frequency. Turn the amplitude up. Vary the frequency f until the string resonates at its fundamental frequency (1 antinode).

Turn the amplitude down a bit when you are fine tuning the frequency – and turn up again while searching for the next resonance.

Repeat with 2, 3 ,4 and 5 antinodes. The deflections gets gradually harder to see. It can be an advantage to look along the string.

Results are recorded in a table like this:

Number of antinodes	f Hz

Measurements – 2

Measure the length L before changing anything.

Repeat the procedure above three-four times with different tension of the string F_S .
You can make do with a few harmonics. Remember to write down the force F_S every time.

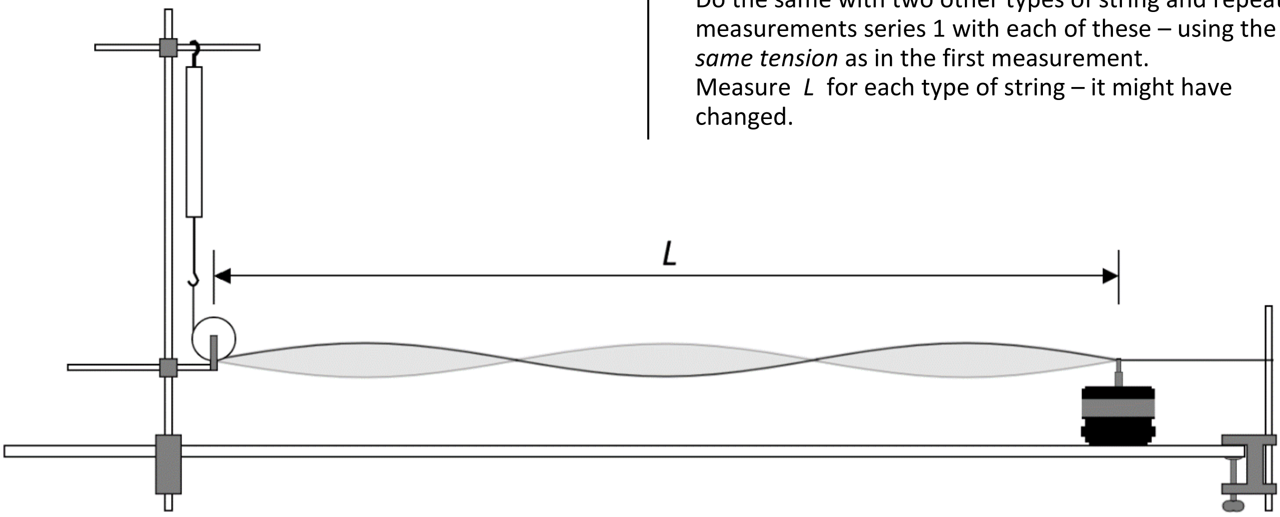
Measurements – 3

Use a 5 m long piece of exactly the same kind of string as used previously – with no knots at all.

Measure the precise length; call it b . (If possible, keep the string as tight as in the experiment.)

Roll up the string and weigh it. Call the mass m .

Do the same with two other types of string and repeat measurements series 1 with each of these – using the *same tension* as in the first measurement.
Measure L for each type of string – it might have changed.



Calculations etc.

For *Measurements – 1*
Calculate the wavelength for each measurement.

Try to formulate a rule for wavelength and frequency (tip: Multiply the two together).

Draw a graph with f along the y axis and the number of antinodes along the x axis. Can you draw a straight line through the data points?

For *Measurements – 2*
(Measurements with same type of thread but with different tensions.)

Calculate, for each value of the tension, an average value of the measured propagation velocity v.

Make a table with columns F_S, v and v^2.

Plot v^2 as a function of F_S.

For *Measurements – 3*
(Measurements with the same tension but with different types of string.)

Calculate, for each type of string, an average value of the measured propagation velocity v.

Calculate, for each type of string, its mass per unit length which we choose to call μ:

$$\mu = \frac{m}{b}$$

Make a table with columns μ, v, $1/\mu$ and v^2.

Plot v^2 as a function of $1/\mu$.

Theory

At resonance, the length of the string will be an integer number of half wavelengths:

$$L = N \cdot \frac{\lambda}{2} \quad N = 1, 2, 3 \ldots$$

N is the same as the number of antinodes. The fundamental corresponds to $N = 1$. Larger values of N correspond to harmonics.

The equation is solved with respect to the wavelength:

$$\lambda = \frac{2 \cdot L}{N}$$

A propagating wave moves just one wavelength λ forwards during one period of oscillation T; hence the velocity v is given by the general expression:

$$v = \frac{\lambda}{T} = \lambda \cdot f$$

We will call the result of this calculation the *measured* speed (λ and f are measured).

In the special case of a wave propagating on a string, it turns out that the velocity is determined by the tension F_S and the string's mass per unit length μ.

We will not derive the expression here but simply present it:

$$v = \sqrt{\frac{F_S}{\mu}}$$

We will call this the *theoretical* value of the velocity.

The expression can be re-written as

$$v^2 = \left(\frac{1}{\mu}\right) \cdot F_S$$

Discussion and evaluating

For *Measurements – 1*
How accurate is your rule of wavelength and frequency?

Describe the features of the graph

For both *Measurements – 2 and – 3*
Compare the graph with the theory. What is the expected slope of the graph? Does it fit reality?

Assess the size of possible deviations, compared with the precision of the measurements.

Teacher's notes

On the general structure of the lab manual

Everybody completes section *Measurements – 1*.

For secondary school this could be sufficiently challenging.

Talented students may complete the next section as well – not necessarily going into a comparison with the theoretical values.

High school students complete all three measurement sections.

Concepts used

- Wavelength
- Frequency
- Resonance
- Harmonics

Mathematical skills (*Measurements – 1*)

- Elementary algebra
- Graph drawing

Mathematical skills (*Measurements – 2 and – 3*)

- Formula evaluation
- Linear functions
- Equation solving

About the equipment

When too high amplitude is used close to a resonance there is a risk that the tension increases or that the oscillations are coupled to resonances in the stand material. This can lead to an unstable amplitude.

Usually it is sufficient to turn down a bit while close to a resonance – but in severe cases you can simply drop measuring the harmonic in question.

Measurements – 1 and *Measurements – 2* were tested with item No 116600 which is a braided Dacron-line.

For *Measurements – 3*, the two other types of line were included.

All measurements agreed with theory with deviations less than 2 %.

The Balalaika photo on page 1 is published by Tomislav Medak under a Creative Commons license.

Source:
http://www.flickr.com/photos/tomislavmedak/3812477660

Detailed equipment list

Specifically for the experiment

250310	Student function generator
… or …	
250350	Function generator (alternatives: 250250, 250150)
218500	Electromagnetic vibrator
208500	Pulley on rod
103840	Dynamometer 5 N

Consumables

116600	Line, braided, 50 m	(approx. Ø 0,5 mm)
799109	Mason cord, 120 m	(approx. Ø 1 mm)
767022	Line, braided, 20 m	(approx. Ø 3 mm)

Standard lab equipment

000820	Retort stand rod, 75 cm
000850	Retort stand rod, 25 cm (2 used)
001600	Table clamp (2 used)
002310	Square boss head (2 used)
105720	Safety cable, silicone, 50 cm, black
105721	Safety cable, silicone, 50 cm, red
140010	Tape measure 200 cm

For *Measurements - 3* you will also need e.g.

102900	Digital scales 300 g / 0,01 g