Tellurium *N*

Table of contents

The teaching units are recommended for students of classes 4 – 8.

The chapter "Further astronomic reflections", which follows each teaching unit, addresses to students and teachers of secondary schools and universities. It should provide, in addition to factual information, an opportunity to infuse these challenging subjects by means of the Tellurium.

General remarks

A Tellurium (after the Latin tellus, the earth) is an apparatus to demonstrate the movements of the earth and the moon. On a lever arm those celestial bodies turn around a source of light which is supposed to represent the sun.

Numerous phenomena in our solar system can be shown on the Tellurium as a three-dimensional model more clearly than with any other medium.

Thus, e.g., overhead-foils are completely unsuited when it is a question of representing the earth as a gyroscope in space. The same applies to the insight into the dynamics of the seasons and times of the day.

In the age of the earth-satellites, the aerial bowls, the astronautics, and a great many students' ideal profession – astronaut – the importance of the outer space for us keeps increasing. Therefore we firmly ought to pay more attention to these processes in school than up to now.

Why are Telluriums at present relatively not much used?

1. During the teacher-training astronomical subjects often are no longer taught at universities respectively do no longer exist in the school -curriculums though, because of the technical development, the outer space gains continuously growing importance for man.

2. The Telluriums so far on the market are less brightly illuminated and do not open the possibilities which this model contains.

The Tellurium N with numerous patented *N*ovelties has been developed by Professor Dr. Jürgen *N*ewig, Geographical Institute of Kiel University in cooperation with Cornelsen Experimenta.[1]
The "Further astronomic reflections" have been collected by Prof. Dr. Hermann König, Kiel.

The Tellurium N is consequently **action-orientated** designed, i.e., the processes do not go off automatically but are executed by schoolboys and schoolgirls showing a great deal of understanding. Thus the lever arm, in order to demonstrate the seasons, is directed by hand by one pupil while at the same time a second pupil gets the earth turned around itself.

As a general rule goes: All processes on the Tellurium are carried out **anti-clockwise**. It begins with the complete turn of the lever arm while focusing the seasons (take hold of the handle) and it also applies to the turn of the earth around itself, the earth's rotation, and finally to the turn of the moon around the earth.

Size of delivery
The **Tellurium N** (item number 31115) comprises the following parts:
Tellurium basic apparatus with Fresnel lens and horizon-disc with shadow-figure

|	
Satellite-stick	
Felt pen, water-soluble	30644
Cleaning-cloth, 2 pieces	18105
Low-pressure Halogen-lamp, 12V/20W (for replacement)	47112
Extension cable with jack/socket, 5 m	
Plug-mains appliance and cable with jack	
Dust cover	311152
Users manual	311156

In addition to teaching unit 7 red and blue plastic modelling material is necessary :
Pack of 10 sticks of Plastilin, assorted colours   70200

Important parts of the Tellurium and their operation
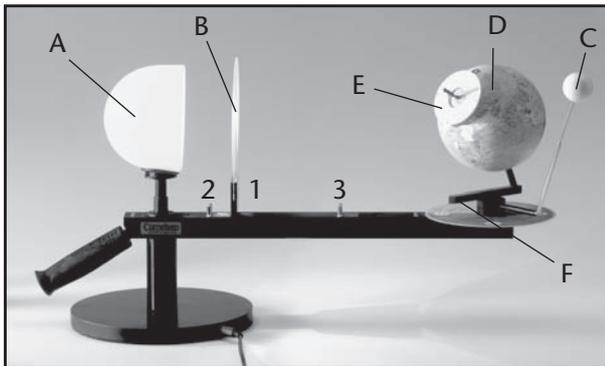
The elements of the lever are:
A   Sun
B   Fresnel lens in normal position (1),
    Moon position (2) and
    sun-point position (3)
C   Moon
D   Earth-globe with telescopic polar axis
E   Horizon-disc with shadow-figure
F   Month-indicator and disc

Sun (A)

For reasons of handiness the sun on this model, in comparison to the 15-cm-globe, is represented too small. Also, its distance to the earth is extremely shortened.

Corresponding to the size of the globe it ought to be at a distance of nearly 2 km and would have the size of a 5-storey building.

Switching on the sun
For switching on the light bulb (only 12V/20W low pressure Halogen-order number 47112) first the jack of the connecting cable is put in the socket on the foot of the Tellurium and then the plug-mains-appliance into a wall socket.(230V/50 Hz)

Change of light bulb
When the light bulb needs to be exchanged, hold it with a cloth.

Tip: The Halogen light bulb develops high operating temperatures. In a cold state, too, the Halogen light bulb may be touched with the help of a cloth only. An explosion-protected low-pressure Halogen light bulb only must be used.

Fresnel lens (B)

Position 1 (**normal position**): The high-quality Fresnel lens provides a very bright directed light similar to the sunlight. All Telluriums so far available send undirected light only, i.e., the light-shadow limit on the globe is not sharp and the intensity of light is only low.

Position 2: When we put the lens on position 2, the cone of light gets larger. Then also the moon will be illuminated.

Position 3: When the lens is put there, then we get a sun-point with a bright halo on the globe.
The sun-point is situated where the sun, at the relevant point of time, is vertical on the earth. The sun-point does not go beyond the tropics.

Tip: Unless indicated differently, the Fresnel lens remains in position 1. In order to change the position, the lens should be taken hold of only by the plastic holder. The lettering on the holder must always point to the sun.

Moon (C)

The moon has been reproduced on the right scale with the earth. Its true medium distance from the earth, however, has been represented far too short. It ought to be about 4.5 m . For this reason the moon always shows itself, when the telescopic rod is not extended, at full moon in the shadow of the earth, which in reality happens only very seldom (Lunar eclipse). In order to demonstrate the full moon in spite of it, we keep pulling the telescopic rod out until the moon is illuminated again by the sun. The nearly circular orbit of the moon around the earth here has been simplified and directed parallel towards the lever arm, i.e., the ecliptic has not been taken into consideration. However, if wanted, it can be simulated for the different phases by pushing down or pulling out the rod holding the moon. This simplification appears to be justifiable because here the only problem is to explain the principle of the moon phases respectively to show the eclipses.

Earth globe (D)

The earth globe, with a diameter of 15 cm, is relatively very big and can be perceived also from farther distance. The high-quality Fresnel lens of a diameter of 16 cm allows to illuminate that size.

The latitude and longitude grit is drawn from 15 to 15 degrees and thus corresponds to the hour-lines.

Globe and disc may be written on only with the encluded or a similar **washable felt pen for overhead projection**.

> **Tip:** For an optimal adhesion of the horizon disc the globe needs to be kept free of dust. Instead of the encluded cloth (before use moisten a bit) also paper tissues or something similar can be used.

Horizon disc with shadow-stick-figure (E)

The horizon-disc with shadow-stick-figure is of fundamental importance for the pupils producing a plastic image, for they will be able, as it were, to put themselves in the figure. At the red line the height of the figure is equal to the length of the shadow.

The horizon disc will always be put on the globe in such a way that the letter N points to the North Pole.

> **Tip:** The horizon-disc always needs to be taken hold of on the edge and to be pressed on the globe while being slightly turned. Please do not put the feet on fibrous cloths as otherwise the adhesion decreases. Should there be any difficulties with the adhesion simply squeeze the adhesive mass in the three feet of the horizon disc with your fingernails a little towards the longer end.

The month-indicator and the date-disc(F)

Below the globe there is a date-disc where, with the help of the pointed side of the month-indicator, you can read the respective month which is just applying for the position of the earth with reference to the sun. That way we cannot do anything wrong on the Tellurium. **Transverse position** of the month-indicator towards the lever arm means equinox in **spring** and **autumn; parallel position** means extreme position in **summer** or **winter** (solstice respectively solstices) In general the days are: March 21, June 21, September 23, December 21. Occasionally (e.g. leap year) the dates are postponed by one day.

> **Tip:** The **handle** should be used for all rotary motions because visibility and stability then are optimal.

Storage and transport of the Tellurium

For storage and transport the plug-mains-appliance can be put in the sockets of the lens-position 3 (see page 4).
To protect it from pollution the Tellurium should always be stored with the dust cover put on.
For transport the Tellurium can easily be held by the **handle**.

Introduction

From one's own shadow to the shadow figure on the globe of the Tellurium

The approach to the proceedings of lighting can best take place via the concrete imagination of the shadow of one's own body. Yet there are numerous pupils who do not know how long or short the shadow in the various seasons is. So before starting work with the Tellurium a teaching unit about the shadow and the corresponding position of the sun is carried out.

We wonder about the length of our shadow in different seasons

1. We imagine standing in the schoolyard at noon in **summer** considering our shadows. Which of you has ever observed how long the shadow at noon in summer with us is?

   • It is shorter than we are tall ourselves.

2. How long is the shadow with us at noon in **winter**?

   • About 4 times as long as we are tall.

3. In **April** and **August** the shadow has a „normal length" with us at noon.

   • The shadow corresponds roughly with the height.

There follows a question going further, to which very often over half of the class do not know the correct answer:
„Are there any regions on earth where people have no shadow at noon?"
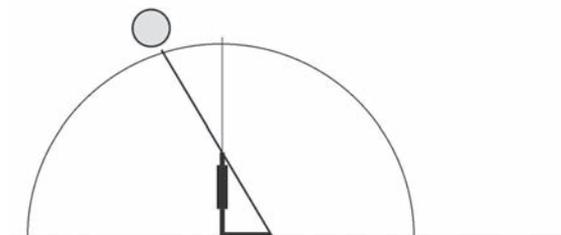Yes, around the equator up to the tropics (see teaching unit 5). The sun then is vertical.

This photo was taken in Tenerife in summer. The island is situated nearly on the Tropic of Cancer. So in Tenerife there is almost no shadow at the end of June.
We can record the result of the occupation with our shadows in different seasons as pictures:

Copy pattern

A: Shadow with us at noon short in summer (end of June)
Shadow = shorter than height of figure (about 61 degrees sun-height in Berlin)
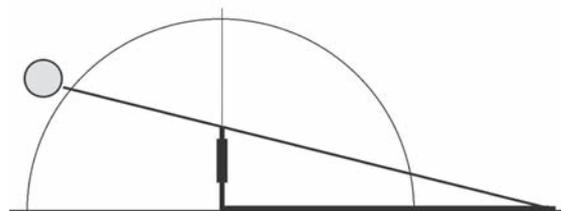
B: Shadow with us at noon long in winter (end of December)
Shadow = about fourfold height of figure (about 14 degrees sun-height)

C: Shadow with us at noon moderately long (April respectively August)
Shadow = height of figure

All these questions can be checked with the help of the Tellurium:

Position A – summer

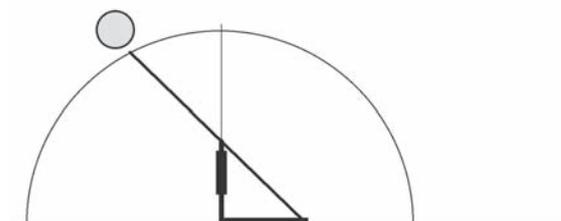We turn the Tellurium with the handle until the point of the red month-indicator points to June 21. Now we have north summer. We turn Europe towards the sun.

We put the shadow-figure on Germany and turn the globe a little to and fro until the shadow is the shortest. The shadow is clearly shorter than the figure, i.e., it moves within the red circle.

Position B – winter:

We leave the shadow figure on Germany and turn the Tellurium with its handle until the point of the month-indicator points to December 21. We now have north winter. The shadow is clearly longer than the figure. The legs (a third of the total length) extend along the entire disc. The whole shadow is about four times longer than the height of the figure.

Position C – April/August:

When we put the point of the red month-indicator on April, then the shadow is as long as the figure. The sun moves until June 21 towards its highest position. After that, in August, they are again on an equal level. We check it by putting the point of the red month-indicator on August.

About the question of shadowlessness

We now move the horizon disc with the shadow figure on the globe to and fro until the figure casts no more shadow. At a certain point of time there is always only one place in the world where that is the case. That place is always between the tropics.

The sunlight falls on one hemisphere of the earth only

With the Tellurium we can see that one half of the globe is always lit by the sun. As the earth rotates, nearly all parts of the earth within 1 day (24 hours) once get on the lit side and once on the unlit one of the globe.

The point of the month-indicator below the globe is put on one of the 2 eqinoxes (around March 21 and September 23). Both poles have a ray of light. In this position can particularly well be demonstrated that a celestial sphere (it also applies to the moon and to other planets) which is floodlit from afar, is always half lit up and half unlit.

Result: The sun is very far away from us. It lights up half of our globe, on the lit-up hemisphere we have day, on the unlit one night.

- We show it by repeatedly turning the globe on the Tellurium.

1. The earth, a gyroscope in space

The Tellurium shows us the movements of the earth around the sun and those of the moon around the earth. Moon and earth are reproduced on the correct scale. The sun, however, ought to be at a distance of nearly 2 km and be as big as a 5-storey house. The moon ought to rotate in the back region of the classroom (distance from the globe 4.5m).

As for the Tellurium those distances are shortened very much.

We put the Tellurium on the teacher's table.

We make a pupil turn the earth several times around the earth's axis.

The notion „earth's axis" will be explained as the line which runs from pole to pole through the centre of the earth.

The arrow on the globe helps us to understand that the earth rotates anti-clockwise.

Result: The earth is a gyroscope in space. It rotates anti-clockwise around the earth's axis.

Now we pull out the extended polar axis as far as possible. We turn the big lever arm until the point of the polar rod is moved as near to the sun as possible. We have reached the optimal position when the red month-indicator below the globe is placed parallel to the lever arm. We have June 21 (see seasons disc).

We realize that the earth as against its orbit around the sun is inclined (about 23.5 degrees inclination of the eclipse).

We make the pupils guess if in their opinion the earth's axis still points in the same direction when we turn the lever arm by 180 degrees (corresponds to half a year).

Tip: Mostly 2 groups of equal size decide for one of the 2 possibilities.

We turn the lever arm with the earth globe until it has completed half a rotation around the sun, anti-clockwise, to be precise.

The lengthened earth's axis still points in the same direction as before, i.e. the earth's axis keeps the direction.
We keep turning until the starting position has been reached again and make sure that the direction of the earth's axis is still right:

The earth's axis always points to the same point of the celestial vault (minor deviations of the axis remain unconsidered on the model). There happens to be a star which is called **Polar Star**.

Position of earth´s axis end of June

Position of earth´s axis end of December

Consequently the Polar Star is the only star which, with the apparent wandering of the stars, always keeps the same place, i.e., always seems to be for us in the same place in the celestial vault, and that is the north. The medieval and antique seafarers already knew that coherence. When they sailed at night, they were able to roughly keep the correct course with the help of the **Polar Star**. We find the Polar Star in the sky when we lengthen the rear of the „Plough" 5 times upwards.
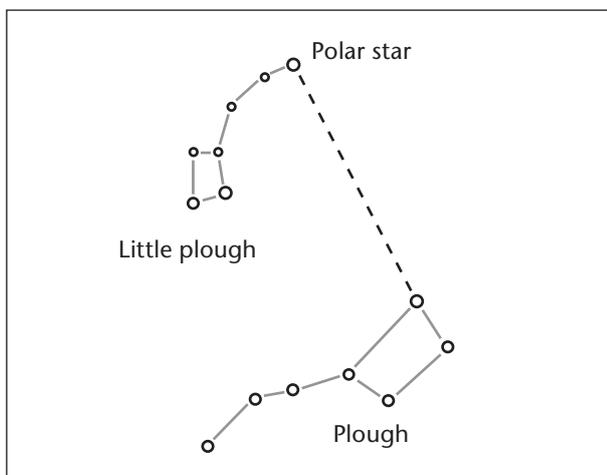
We record the following basic under-standing on the blackboard:

> The earth behaves like a gyroscope in space. The earth's axis always points in the same direction. In reality the lengthening of the polar axis points to a star which happens to be in that place and so has got the name of „Pole Star" (or polar star).

Further astronomic reflections

The earth is a gyroscope in space, turning – viewed from the north – counterclockwise once per day around its axis. As a result of the centrifugal force of this rotation the earth has a bulge near the equator. The Geoid – figure of the earth – is approximated by an ellipsoid with one axis in the direction of the North Pole to the South Pole of 6,356.8 km and two equal axes in the equatorial plane of 6,378.1 km. The earth rotates once per year around the sun, also counterclockwise when viewed from the north. This movement occurs in a plane, the **ecliptic**; it is not circular but of ellipsoidal form according

to Kepler's laws. The excentricity of earth's orbit is e = 0.01671. Earth's axis and the axis perpendicular to the ecliptic are inclined by an angle of $\varepsilon = 23.44°$. This is also the inclination of the equatorial plane and the ecliptic. At the spring and autumn equinox the earth passes the nodal line, i.e. the line of intersection of both planes.

Since earth's mass distribution is uneven in view of the equatorial bulge, the gravitational attraction of sun and earth tries to tilt the equatorial plane into the ecliptic. However, just as with a spinning gyroscope, earth's rotational axis moves in a perpendicular direction. Thus earth's axis spins clockwise once around the celestical north pole in about 25,800 years. This results in the **precession of the equinoxes**: the spring equinox moves backwards with respect to the fixed stars by 1° every 72 years. Right now the spring equinox is in the constellation of fishes, it will move into the constellation of aquarius in about the year 2600 ("age of aquarius").
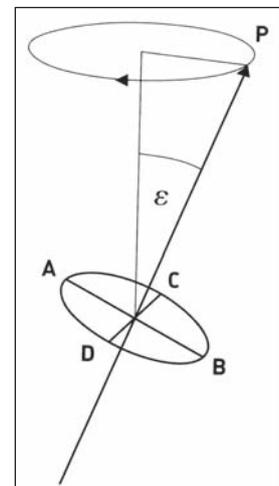
Precession of the earth's axis, equatorial plane of the earth in ACBD

Due to the precessional movement of earth's axis, the celestical north pole moves closer to the pole star Polaris until 2100 and away from it afterwards. About 4,500 years ago – during the construction of the Egyptian pyramids – the northern direction of earth's axis showed a deviation of around 25° from the pole star, right now it is only ¾°. Smaller movements of earth's axis result from gravitational forces of the moon, planets and earth: the nutation is a result of lunar forces. The obliquity of the ecliptic of 23.44° now is changing within certain limits, namely between 22 ½° and 24 ¼°, it is slowly decreasing in our times.

2. Day and night
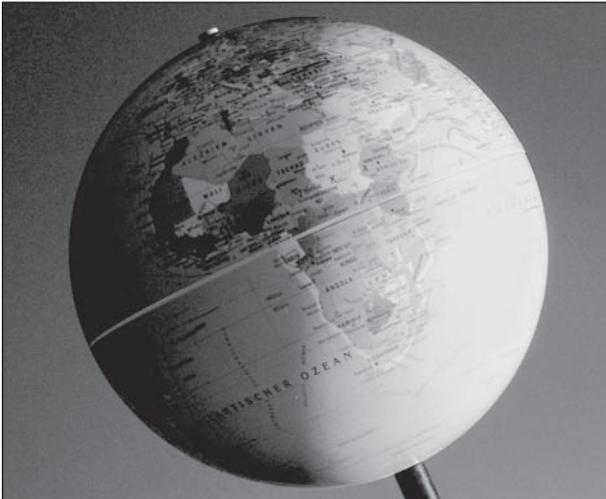
1. Day-and nightside

Just one half of a ball is lit when the source of the light is far away. As the sun has a distance of millions of kilometers from the earth, it always lights one hemisphere of the earth.

We have to make it clear to the pupils that this conformity to a law applies to all corresponding celestial bodies of spherical shape, thus, e.g., to the moon, but also to our neighbouring planets. The same way as the moon sometimes is very brightly or practically hardly visible, also the planets change their brightness.

Result:

• We call the relevant sunlit side the **dayside**.

• The side not sunlit is the **night-side**.

That is shown on the Tellurium.

> **Tip:** In the course of 24 hours every part of the earth experiences a change of day and night with the exception of the pole region, where are particular conditions.

• From the unlit side we cannot see the sun.

That is demonstrated with the help of the shadow-stick-figure.

• From the lit side we can always see the sun unless it is hiddenby clouds.

As before, that is demonstrated with the help of the shadow-stick-figure.

2. The earth turns around itself (earth rotation anti-clockwise)

Day-and nightside alternate because of the earth inherent rotation, **the earth rotation**. It turns around itself once in 24 hours.

A pupil performs several full turns of the globe (as the arrow shows, anti-clockwise). It is observed that the light/shadow limit always seems to remain in the same place though the appearance of the earth turns off below it.

In the transitional zones between light and shadow we observe the twilight. The shadows at sunrise and sunset are very long.

That is shown with the help of the shadow-stick-figure on the Tellurium.

3. How do we make out on the globe that it is noon?

We put the Tellurium on north summer (June 21). The point of the red month-indicator below the globe points to the sun.

With a felt pen we mark the meridian (the midday line) that runs through Berlin 15 degrees east of Greenwich near London).

We put the shadow-stick-figure on Berlin. Slightly turning the globe to the right and to the left we can see when the shadow is the shortest. We now have noon.

So at noon we cast the shortest shadow, and that one, on the north hemisphere outside the tropic, points northwards.

4. All-year equal lengths of day and night at the equator (cp. teaching unit 7)

Tip: Because of the earth-rotation a person at the equator covers the length of the equator of 40,000 km, i.e., the circumference of the earth, in 24 hours. That is p.h. 40,000 km ÷ 24 = 1,666.70 km. But he does not notice the speed because it is constant. A person at the pole only turns once about himself in the same time, practically covers no distance.

The shadow-stick-figure is put on the equator. The red month-indicator is placed parallel to the lever arm (June 21 -north summer). A red plasticine strip is put along the lit area, a blue one on the unlit one. Both are stuck on the blackboard. Both are of the same length. (cp. teaching unit 7)

Because of the high rotary speed of the equator the sun turns very fast into the light or into the shadow. The everywhere equally wide belt of twilight crosses the region of the equator in a very short time. Therefore the **phase of twilight** at the equator is extremely short. At half past five p.m. it is still bright and the sun is clearly above the horizon. At half past six p.m. the night sky is already dark.

Repeat questions

How many kilometers does a person cover in the day time at the equator because of the inherent rotation of the earth, i.e., in the sunlit region and how many kilometers at night?

- In both cases 20,000 km.

How much time does he need for it?

- In both cases 12 hours, i.e., altogether he covers 40,000 km in 24 hours.

Further astronomic reflections

The word "day" is used in two meanings: first, a (tropical) day of 24 hours, the mean period of time between two transits of the observer's meridian by the sun and second, the period of daylight at a particular location on earth in contrast to the night, the period of darkness.

The meridian marks the highest daily position of the sun in the sky. A **tropical** day is longer than a **sideral day** by about 4 min which is the time after which a point on earth points to the same position (longitude) on the celestical sphere. Since the earth has by that time turned around the sun by about 1°, the earth has to turn by this additional degree until the sun passes the meridian again. 1° corresponds to $1/360$ (tropical) day = 4 min.

The circumference of the earth at the equator is $2\pi \times 6,378.1$ km = 40,075 km, the rotational speed of the earth at the equator is hence about 1,670 km/h. The circle of latitude for Berlin at about 52.5° N has a circumference of only 24,370 km, the rotational speed there is thus only 1,015 km/h.

On other planets, "days" may take much longer: Venus needs 243 earth-days to turn once around its axis, Mercury 59 earth-days whereas Mars shows with 24½ h about the same rotational period as the earth.

In summer on the northern hemisphere, earth's axis is tilted towards the sun. Therefore in latitude φ with $0 < \varphi < 90°$ more than one half of the parallel of latitude φ is illuminated by the sun: the day is longer than 12 h, the night shorter than 12 h. This effect is more pronounced further north. North of the polar circle ($\varphi > 66.56°$) at the summer solstice there is no night. At the winter solstice the reverse happens: north of the polar circle the polar night lasts 24 h. Also at our latitudes in central Europe the differences in the length of daylight are remarkable: at latitude $\varphi = 52.5°$ N (Berlin), daylight occurs for 16h 50min at the summer solstice and for only 7h 40min at the winter solstice. On June 20, a (sunlight) day is 40 min longer in Flensburg than in Munich. The sun

rises in the east and sets in the west during the time of the spring and autumn equinox. In mid-June, however, the sun rises in the north-east and sets in the north-west. The length of daylight T is given approximately by the formula

$$T = \frac{24}{180} \text{ arc cos}(x) \text{ hours}$$

$$x = \frac{\tan \varphi \cos \psi}{\sqrt{\cot^2 \varepsilon + \sin^2 \psi}}$$

where φ denotes the latitude, ψ the angle in the ecliptic measured from the winter solstice, $\varepsilon = 23.44°$ is the obliquity of the ecliptic and arc cos(x) is an angle between 0° and 180°. For |x| >1 there is 24 hours daylight (x < –1) or night (x > 1). Thus the length of daylight depends on the latitude and the season which affects the tilt of earth's axis towards the sun.

The Formula actually gives values which are about 15 min too short since it does not take into account the refraction of light in the atmosphere and the finite diameter of the sun of about ½°: the sun is seen already before "geometrically rising" and still seen after "geometrically setting". **Civil twilight** is the period of time during which the sun is between 0° and 6° below the horizon. Since the zone of twilight has the same width, the period of twilight is much shorter at the equator than in central Europe. There is also a slight variation of twilight with the seasons. At latitude $\varphi = 52.5°$ (Berlin) civil twilight lasts for about 58 minutes before sunrise and after sunset at the beginning of summer, for about 48 min at the beginning of winter and only 40 min during the spring and autumn equinoxes. At the equator, the phase of civil twilight lasts only 26 min on June 20 and December 22 and 24 min on March 21 and September 23; it is thus much shorter near the equator.

3. Midday line and division of hours

1. All places on a meridian have noon at the same time

The Tellurium is put on September 23.

With the felt pen we draw a line from the North Pole via Berlin further on to the south as far as beyond the equator. That line marks the 15th degree of longitude east of Greenwich near London, which almost runs across Berlin.

We put the shadow-stick-figure on that line, in fact exactly on the equator. Slightly turning the globe to and fro we get the point where the figure casts no longer any shadow, here now is midday. The sun is vertical.

- It is above the equator, 15 degrees of east longitude, a little east of Libreville, the capital of Gabun.

We leave the globe exactly in that position (it will be best if one pupil holds the globe and another one pushes the shadow-stick-figure northwards to Berlin). We pay attention that the north direction on the disc points to the North Pole. The shadow now points exactly northwards. So the sun is in the south and we have noon in Berlin. The shadow reaches a bit beyond the red line, i.e. it is a little longer than the figure.

If we now put the figure further southwards on Rome, the shadow becomes much shorter.

As a check we go again on the equator near Libreville.

The next place we look up is Walfisch Bay, a town in Namibia. It is situated at the Tropic of Capricorn.

Tip: All places on a midday line have noon at the same time as the name already says. On flights we can travel thousands of kilometers to South Africa, but need not adjust our watches if we remain on the meridian. (Slight differences are possible, e.g., if in summer no summertime adjustment takes place.)
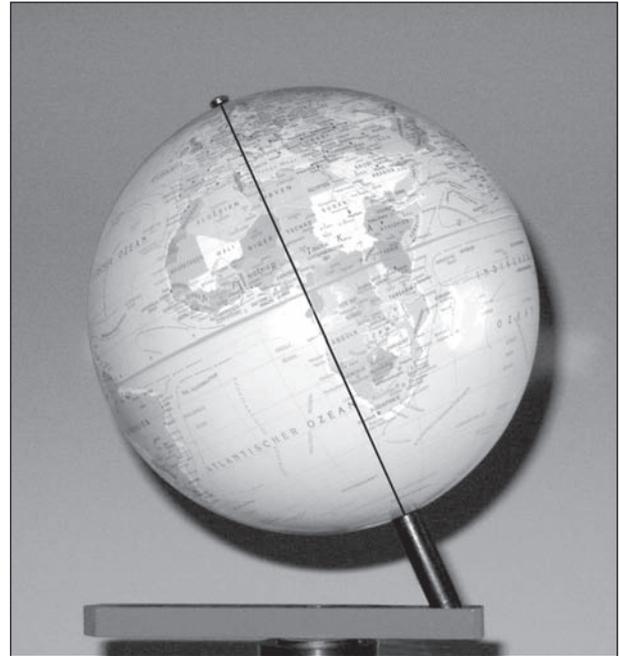
2. On the south-hemisphere the sun shines from the north(in the tropics, however, only temporarily)

Let us consider the shadow near Walfisch Bay a little more closely. It is clearly shorter than the one in Berlin for the place lies nearer to the equator.

But above all: **The shadow in Walfisch Bay in southern Africa is directed to the south. Here the sun shines in the north summer from the north.** Further down to the south it even shines all year from the north, e.g. in Johannesburg or Capetown. The cardinal points at sunrise and sunset, of course, remain east and west, like with us, for after all the earth turns in one direction only.

3. The hour as the 24th part of a day

The Tellurium remains put on September 23 (equinox). The red month-indicator is crosswise to the lever arm.

The distance between 2 midday lines or meridians is called **degree of longitude**. The degrees of longitude are counted twice up to 180 degrees(east and west length of the prime meridian of Greenwich near London), thus altogether 360 degrees for the full circle. As the earth in 24 hours turns once around itself, 360 degrees correspond to 24 hours.

Put differently: **One hour is the 24th part of a full earth rotation**

- A full earth rotation is shown at the Tellurium

The half of the day counts 180 degrees of longitude, and so does the half of the night.

They are counted out corresponding to the grit lines from 15 to 15 degrees on the globe.

If half of the day comprises 12 hours, it means that (180:12) **15 degrees fall to one hour**.

So, through its inherent rotation opposite the sun, the earth covers a strip of 15 degrees per hour, which is very large (1,667 km) at the equator and becomes more and more narrow in the higher latitudes.

4. The distribution of the hours across the earth

We draw a black line on the prime meridian from the North Pole via London as far as the equator. We put the figure on the equator and turn the globe a little until the figure has no longer any shadow.

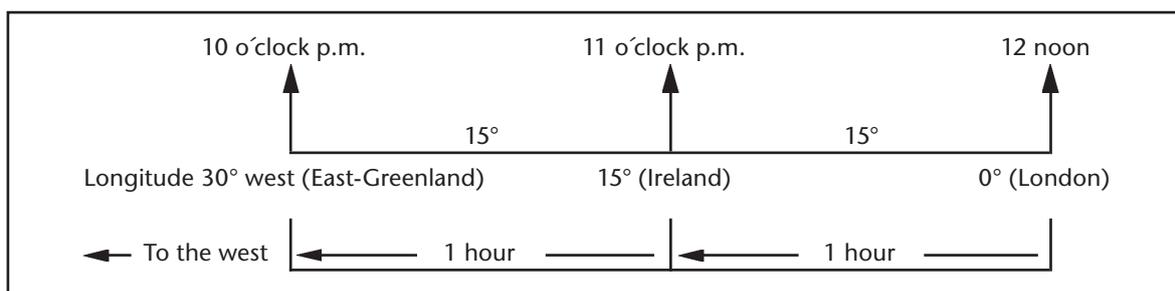- Now there is noon on the whole prime meridian.

A pupil writes "12 o'clock" beside London.

We now consider how late it is in Berlin. As the sun turns anticlockwise, Berlin must have had noon. Berlin is about 15 degrees east of London, i.e., 1 hour shifted. So in Berlin it is now 13 hours. We write „13" hours at the 15th meridian near Berlin. We now proceed hour by hour eastwards (14,15,16 etc). In Bangladesh it is already 18 hours, in Japan 21, and on the 180 meridian, the date line, we have 24 hours.

In the same way we proceed on the western hemisphere. In Ireland, on 15 degrees west, we have 11 o'clock p.m., on East-Greenland 10 o'clock, in New York 7 o'clock, in California 4 o'clock, and at the date line (180 degrees west) 0 hours.

5. The date line

Tip: At the 180th meridian meet 2 days. It leads to a date difference of 1 day, which becomes apparent on crossing the date line. From east to west we jump a day, from west to east we count a day of the calender twice.

| 10 o´clock p.m.	11 o´clock p.m.	12 noon
↑	↑	↑
15°	15°	
Longitude 30° west (East-Greenland)	15° (Ireland)	0° (London)
← To the west	← 1 hour	← 1 hour

Further astronomic reflections

One hour is the 24th part of a tropical day. In antiquity and still in the middle ages, when there were no precise clocks, the hour was the 12th part of the daylight period at a particular location on earth. The length of an hour thus depended on the season and the latitude. Further, any city hat its own local time depending on the meridian, i.e. the longitude of the city. The sun passes the **meridian** on a particular day for a specific location on earth when the sun attains its highest position in the sky; the shadows of persons are the shortest at this time. The multitude of local times proved to be a nuisance in the 19th century when the railroad systems expanded. Around 1840 England introduced a common time zone based on the local time at London/Greenwich. Before that, the local time e.g. in Bath was 10 min behind the one in London. Similar attempts with time zones started in the USA a few years later. In the year 1884 an international conference in Washington fixed the time zones still used today. They are generally based on the local times of the closest Multiple-of-15°-Meridian, with the 0°-Meridian passing Greenwich. The central European time is based on the meridian of 15° East near the German-Polish border. In locations of 10° East – e.g. in Hamburg – the sun reaches its highest point in the sky only 20 min later than in points of 15° East on which the time zone is based. The 15°-time zones have a width of 1,670 km at the equator – 1/24 of the circumference of the earth of 40,075 km. At the latitude of 52.5° (Berlin) the time zone has a width of only around 1,000 km since the parallel of latitude 52.5° has a smaller circumference.

The earth moves along an elliptical orbit around the sun, with the sun in one of the two foci of the ellipse. A smaller distance of the earth to the sun means a higher orbital speed (Kepler's laws). The earth reaches the closest point to the sun each year around January 4, and is farthest from the sun around July 5. These dates slowly move within centuries to later dates in the year due to the precession of the equinoxes. The angular speed of the earth around the sun is even greater in the (northern) winter than in the summer: the earth has to turn around its axis slightly more to again reach the meridian than for the yearly average day. "Days" in January thus may be up to 8 sec larger than mean solar days. In summer this situation reverses. Moreover, the tilt of earth's axis results in another variability of the "day" measured from meridian to meridian, depending on the season: this time varies up to ± 20 sec compared with a mean solar day. This is since the projection of an angle α in the ecliptic with respect to spring equinox into the equatorial plane (obliquity of $\varepsilon = 23.44°$) yields a slightly differing angle unless $\alpha = 0°$, 90°, 180°, 270°. Both effects combined are represented cumulatively in the **"equation of time"**: In December "days" may be up to 28 sec longer than a mean solar day, in September up to 22 sec shorter.
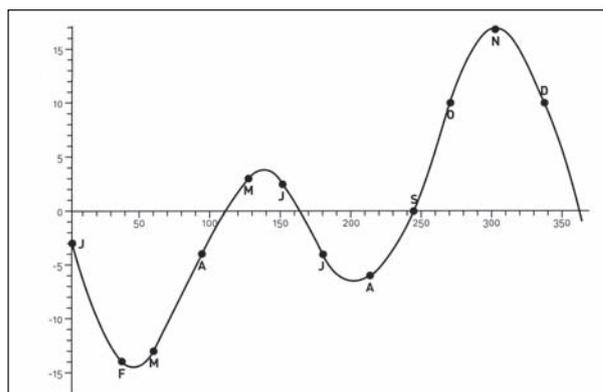
The Equation of time: Apparent time minus Mean time

To have days and hours of equal length, in 1900 the **tropical day** was defined and introduced which is the yearly average (over several years around 1900) of meridian-based solar days. This mean solar time may differ from the apparent solar time – measured by the sun's passing of the meridian – by up to 16 min; this is the cumulative effect of "days" longer by up to 28 sec or shorter by up to 22 sec than averages ones. The equation of time describes this effect, it is defined as the difference of apparent and mean solar time. The maximum of 16 min is reached on November 4, the minimum of –14 min on February 12. In Hamburg e.g., the sun reaches its highest location in the sky on February 12 only at $12^{00} + 20$ min (10° E instead of 15° E) + 14 min (equation of time) = 12^{34}.

4. Polar day and polar night

The Tellurium is put on north summer: The point of the red date-indicator below the globe points to June21.

The North Pole is inclined towards the sun. That way the northern hemisphere gets more sunlight in this position than the southern hemisphere. Therefore the sun lit part of the northern hemisphere reaches beyond the pole.

We turn the earth by 24 hours, i.e., a full rotation, and are observing the light-shadow-line at the North Pole as well as the North Pole itself. The North Pole lies in the light during the time of the full rotation. So the sun does not set as often as we might repeat the rotation.

One pupil keeps the enclosed washable (!) felt pen vertical on the light-shadow-line, another one makes the globe rotate underneath it. That way results a circular line. It corresponds to the **polar circle**. Within the polar circle there are days in the year on which the sun neither rises nor sets. At the pole that time lasts half a year, at the polar circle it is true for one night only.
So, in the north summer, i.e., at the now set time, at the pole the sun does not set for half a year. That time begins a quarter of a year before June 21 and ends a quarter of a year after it, i.e., at the equinoxes in March and September. During that time at the pole shines the **midnight sun**.

On the southern hemisphere, where there is now winter, the sun does not rise at the pole for half a year. Here we have the **polar night**.

We now turn the Tellurium by 180 degrees, i.e., by a semi-circle following, with the polar rod pulled out, the direction of the pole. It still points to the same place in space. But now the pole is turned away from the sun and the South Pole is turned towards the sun.

Therefore we now have polar night in the region of the North Pole. We turn the globe several times around itself and see that the sunlight does not reach this region.

The North Pole region is also called the **Arctic**, the South Pole region the **Antarctic**. In the Antarctic we now have midnight sun.

Further astronomic reflections

At the beginning of the northern summer on June 21 the earth's axis (North Pole to South Pole) is tilted by $\varepsilon = 23.44°$ towards the sun. North of the **Polar Circle**, the circle of latitude $\varphi = 90° - 23.44° = 66.56°$ N, the sun shines for 24 hours a day. At the north pole, this polar day lasts for half a year, from the spring to the autumn equinox. North of the polar circle the midnight sun shines for certain periods of the time between the spring and autumn equinox, all the longer the further north the location is. Close to the south pole, between 66.56° S and 90° S during this time the polar night occurs.

At the beginning of the northern summer at the polar circle the sun reaches a height of 46.9° above the horizon when passing the meridian; the midnight sun shines from the north, the sun being on the horizon. At the north pole, the sun stays on June 21 all day at a height of 23.44° above the horizon. In latitudes in between, e.g. $\varphi = 75°$ N, similar facts hold: the sun reaches its highest point of $23.44° + (90° - 75°) = 38.44°$ over the southern horizon, and as midnight sun its lowest point of $23.44° - (90° - 75°) = 8.44°$ over the northern horizon.

Due to twilight, the sky is not completely dark for 24 hours also south of the polar circle in summer. Considering the period of civil twilight, the sun being above 6° below the horizon, this happens for 24 hours on June 21 north of latitudes 60.56° N. For nautical twilight – sun up to 12° below the horizon – this is even true north of 54.56° N.

5. The Tropics of Cancer and Capricorn and the tropics

For this teaching unit the Fresnel lens is to be put on position 3 „sunpoint".

We turn the lever arm till the month-indicator points to June 21 (north summer).

We put the horizon-disc with the shadow-stick exactly on the sun-point and see that the figure casts no shadow.

• In a simple way the sunpoint always shows us the place where the sun is vertical on the earth.

We put the horizon disc aside und put a felt pen on the sunpoint. A second pupil is turning the earth under the sunpoint around itself until we get a closed circle.

We now turn the lever arm by 180 degrees. Below the point of the month-indicator appears June 21. We put the felt pen again on the sunpoint and below it keep turning the earth till we get a second circle at the same distance from the equator.

The two circles are called **the Tropics of Cancer and Capricorn** because the sun cannot proceed further polewards but, after its highest position, turns again in the direction of the equator.

• The northern one is the Tropic of Cancer, the southern one the Tropic of Capricorn called after the signs of the zodiac.

The region between the 2 circles is called **the tropics**.

• In the tropics (apart from the lines of the Tropics of Cancer and Capricorn) the sun, in the course of the year, does not shine from one direction but changes it. At the equator the sun shines for half a year from the north (between March 21 and September 23) and half a year from the south. But it always rises in the east and sets in the west (also see teaching unit 3.2).

We now turn the lever arm by 180 degrees, i.e., we put the Tellurium on June 21.

We turn the globe till the prime meridian has been put on the sunpoint. We mark that place as a cross with the felt pen. We now turn the Tellurium by 1 month further on to the end of July. There we stop and make another cross on the sunpoint on the globe. The same happens for all the further months including May.

We connect all points with each other and get a curved line, which swings between the 2 tropics around the equator.

We visualize the procedure again by carrying out a full year's turn with the Tellurium and consciously consider **the apparent turning of the sun** at the Tropics of Cancer and Capricorn.

Result: The sun can be vertical only in the region between the Tropics of Cancer and Capricorn, not beyond them.
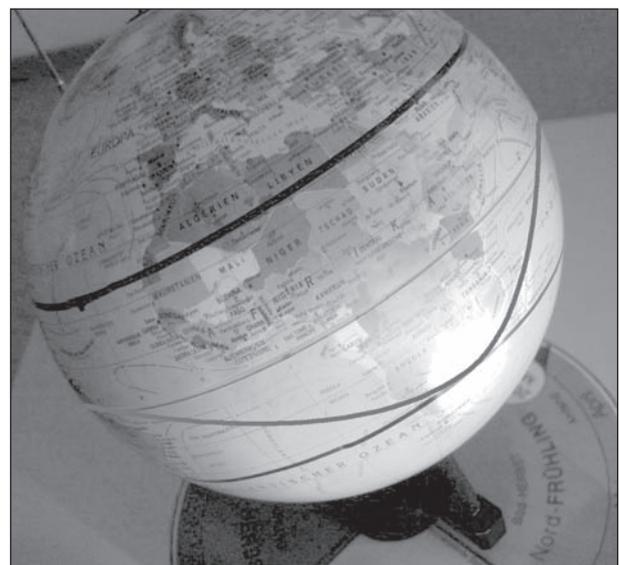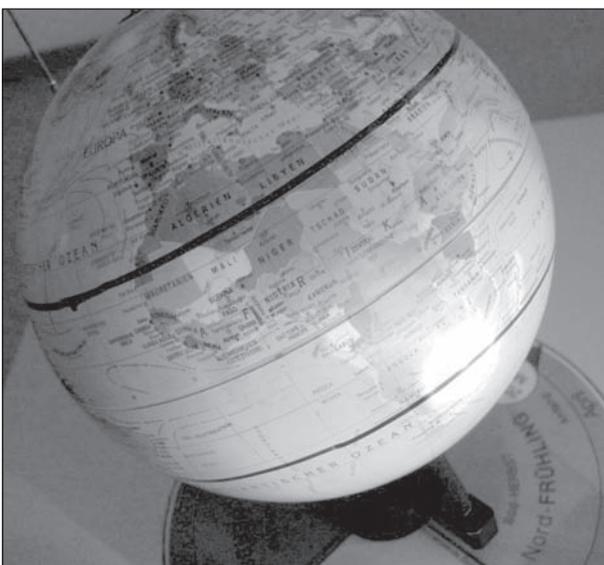
Further astronomic reflections

At the beginning of the northern summer on June 21 the sun stands vertically over the circle of latitude 23.44° N. This is the **Tropic of Cancer**, the earth's axis being tilted by $\varepsilon = 23.44°$ towards the sun. Similarly, at the beginning of winter the sun shines directly overhead of the **Tropic of Capricorn** at latitude 23.44° S. Only in the Tropics, the area between the Tropic of Cancer and the Tropics of Capricorn, the sun may be shining directly vertically from the sky. In latitudes between 23.44° N and 23.44° S this happens exactly twice per year, at the equator e.g. at the spring and the autumn equinox. In the tropics, the sun may shine from the north or the south depending on the season.

Since the **obliquity of the ecliptic** of $\varepsilon = 23.44°$ N right now varies within certain bounds over millennia, the tropics and the polar circles slightly change. Around the year 7500 BC the obliquity of the ecliptic reached a maximum of 24 ½°, around 12000 AD it will reach a minimum of 22 ½° and then start to increase again. These variations result from gravitational effects of the planets on the earth geoid. The variation of the obliquity of the ecliptic has a mean period of 41,000 years. The nutation is an additional periodic change of the tilt of earth's axis, of much smaller size, caused by moon's gravitational effect on the geoid. The main period there is 18.6 years, the period of rotation of the nodal line of the moon; this nodal line is the intersection of the plane of the moon's orbit with the ecliptic. Even if the Tropic of Cancer has been given a fixed mark as near Assuan/Egypt, it slowly moves, right now southwards about 14 m per year.
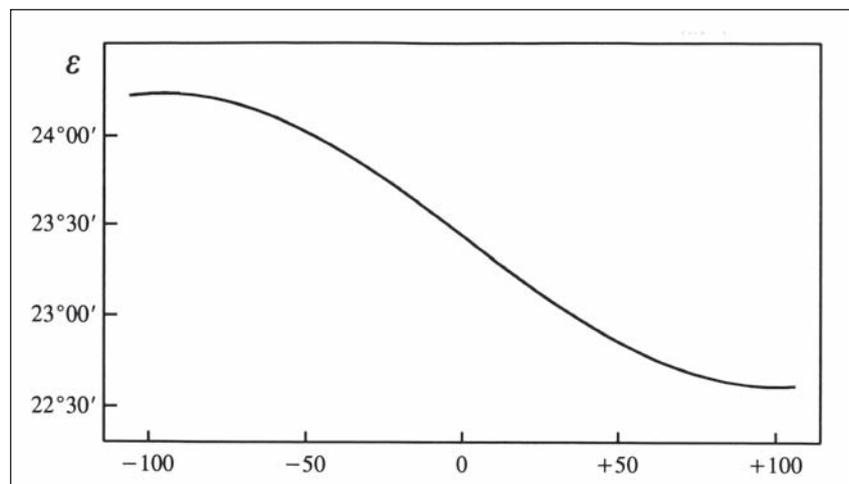
Obliquity of the ecliptic in centuries from the year 2000
(taken from J. Meeus: *Astronomical Algorithms*)

6. The seasons

1. North summer (south winter)

We begin with the north summer, i.e., the month-indicator is parallel to the lever arm, we have one of the two extreme seasons. The red point of the date-indicator points to June 21(solstice).

The northern hemisphere is turned towards the sun (at an angle of 23.5 degrees – the slope of the ecliptic), which can particularly well be made out when the pole-axis rod is pulled out.

We shift the shadow-stick-figure till it shows no longer any shadow. Now we have noon and are on the Tropic of Cancer.
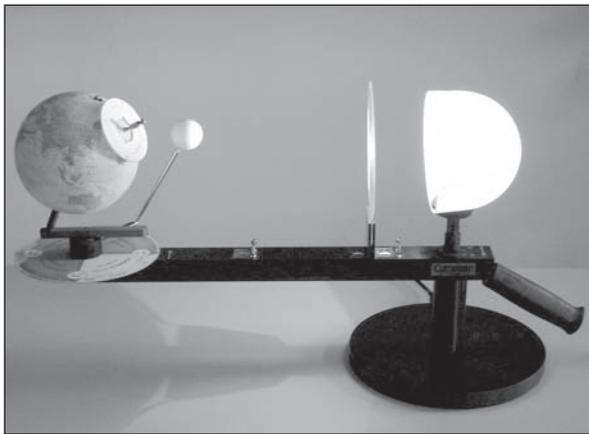
North summer: The axis of the earth is inclined towards the sun.

Result: On June 21 (north summer) the sun is vertical above the Tropic of Cancer. The northern hemisphere is altogether favoured by the sunlight.

2. North autumn (south spring)

We turn the Tellurium by a quarter turn – anti-clockwise. It is now on September 23. Caused by leap-years and other reasons the date can occasionally shift by 1 day.

• We place the shadow-figure in such a way that it does not cast any shadow. It is on **the equator.**

The northern and southern hemispheres are sunlit in the same way.

North Pole and South Pole are at an equal distance from the sun, i.e., the sloping position of the earth's axis now runs crosswise to the sun.

Both poles are at an equal distance from the sun. The sunlight grazes both poles. Everywhere in the world there is now equinox, i.e. 12 hours day and 12 hours night.

It is **north autumn** respectively south spring.

Result: On September 23 (north autumn) the sun is above the equator. It is equinox. Everywhere there are 12 hours day and 12 hours night.

3. North winter (south summer)

After another quarter turn of the Tellurium we come to December 21 (winter solstice) and with it to the second extreme season, the **north winter**. The sun is now vertical above the Tropic of Capricorn as we can see with the help of the shadow-stick-figure.

North winter: At noon the sun is vertical above the Tropic of Capricorn

Result: On December 21 (north winter) the sun is vertical above the Tropic of Capricorn. The southern hemisphere is altogether favoured by the sunlight.

4. North spring (south autumn)

After another quarter turn we come to **north spring**. The point of the date-indicator points to March 21. Again, like in autumn, both poles are grazed by the sun. We now have the second equinox of the year.

We put the shadow-figure again on the place where it does not cast any shadow. It is again on the equator, i.e., at the equator the sun is vertical every half year. In the other regions, too, between the Tropics of Cancer and Capricorn, the sun is vertical twice a year, but not at the same intervals.

Result: On March 21 (north spring) the sun is above the equator. It is equinox. Everywhere we have 12 hours day and 12 hours night.

At the Tellurium we make sure that, because of the inclination of the earth's axis, the northern hemisphere gets more sunlight than the southern hemisphere on June 21. We turn the lever arm by 180 degrees, on December 21. So both halves of the earth, in the course of the year, are equally sunlit. In that way heat and cold on the earth spread better than in case the earth's axis were vertical (to the path of the earth around the sun).

By an accident of nature the axis of the earth is **inclined** by 23.5 degrees with regard to the perpendular to the path of the earth around the sun. That way there is a relatively wide area of sufficient warmth in which human, animal and vegetable life is possible.

Result: Through the inclination of the earth's axis nature provides more living space (biosphere) for man, plant, and animal.

Further astronomic reflections

The seasons originate from the changing angle of the sun's position in the sky when passing the meridian over the year. This is a result of the inclination of the equator and the ecliptic by $\varepsilon = 23.44°$. At the beginning of the northern summer on June 21 earth's axis is tilted towards the sun and the Tropic of Cancer, $\varphi = 23.44°$ N, is hit vertically by the sunshine at midday. In Berlin, the sun then reaches a height of $90° - 52.5° + 23.44° = 60.94°$ N which is the maximal possible value there. The influx of energy per square meter in Berlin is only 13 % less than if the sun would be vertically overhead ($\sin 60.94° = 0.87$). At the winter solstice in December 22, the sun attains its midday height of only $90° - 52.5° - 23.44° = 14.06°$, the energy influx being 76 % less compared with the sun shining vertically overhead ($\sin 14.06° = 0.24$). Comparing the winter and summer energy influx in Berlin at midday, the winter influx is only about 28 % of the one in summer ($\sin 14.06° \div \sin 60.94° = 0.28$). At the spring and autumn equinox, the energy influx at midday is about 39 % less in Berlin than at the equator ($\sin 37.5° = 0.61$). If there would be no inclination of the equator and the ecliptic, the influx of solar energy would be the same all year long, and there would be no seasons.

At the time of the spring equinox ($\approx$ March 21) and the autumn equinox ($\approx$ September 23) earth in its orbit around the sun passes the nodal line, the line of intersection of the equator and the ecliptic. The sun is then vertically overhead at the equator at midday. During the time of the summer solstice ($\approx$ June 21) the sun is vertically overhead the Tropic of Cancer, at the winter solstice ($\approx$ December 22) vertically overhead the Tropic of Capricorn. In these points of time, the sun is at a standstill on its way north or south and then reverses.

According to **Kepler's first law**, the earth moves in an elliptical orbit around the sun, the sun being in one of the foci of the ellipse. The mean distance of the earth from the sun is 149.6 million km (1 AU = 1 astronomical unit). The shortest distance each year is reached at the beginning of January (≈ January 4) with 147.1 million km, the farthest at the beginning of July (≈ July 5) when it is 152.1 million km. Thus the excentricity of the ellipse is e = 2,5 ÷ 149.6 = 0.01671. The energy influx from the sun on earth is about 7 % higher in the northern winter (January 4) than in the summer, since the planar ratio is $(152.1 \div 147.1)^2 = 1.07$. The cooling in the northern winter is thus slightly less than it would be in the case of a circular orbit of the earth around the sun, the cooling in the southern winter is slightly higher.

According to **Kepler's second law**, the radius vector Earth – Sun passes equal areas in equal times. Since the distance Earth – Sun is smaller in winter than in summer, the orbital and angular speed of the earth around the sun is higher in the winter compared with the summer. The winter half year (autumn to spring equinox) is thus by 7 ½ days shorter than the summer half year (spring to autumn equinox), on the northern hemisphere. On the southern hemisphere the effect is just the opposite. The angular speed on January 4 is by about 7 % higher than on July 5, since $((1 + e) \div (1 - e))^2 = 1.07$. As for the length of the seasons on the northern hemisphere, the following is true: Spring 92.75 days, Summer 93.66 days, Autumn 89.83 days and Winter 89.00 days. Due to the precession of the equinoxes, these values change over millennia. Right now, the northern winter is shorter and milder than the southern winter.

Our calendar is based on the **Tropical Year**, reflecting the cyclical character of the seasons: the tropical year is the mean value of the periods from one spring equinox to the next, from one summer solstice to the next etc. This year lasts for 365.2422 days = 365 d 5 h 49 min. The mean length of the Gregorian calendar is 365.2425 days, being on average 27 sec per year too long. One has to distinguish the Tropical Year from the **Sideral Year**, which is the period of time after which the earth reaches again the same longitude with respect to the fixed stars. Due to the precession of the equinox, the sideral year is 20 min longer than the tropical year, its value is 365.2564 days.

The seasons and inclination of the earth's axis

L = Angle of seasons
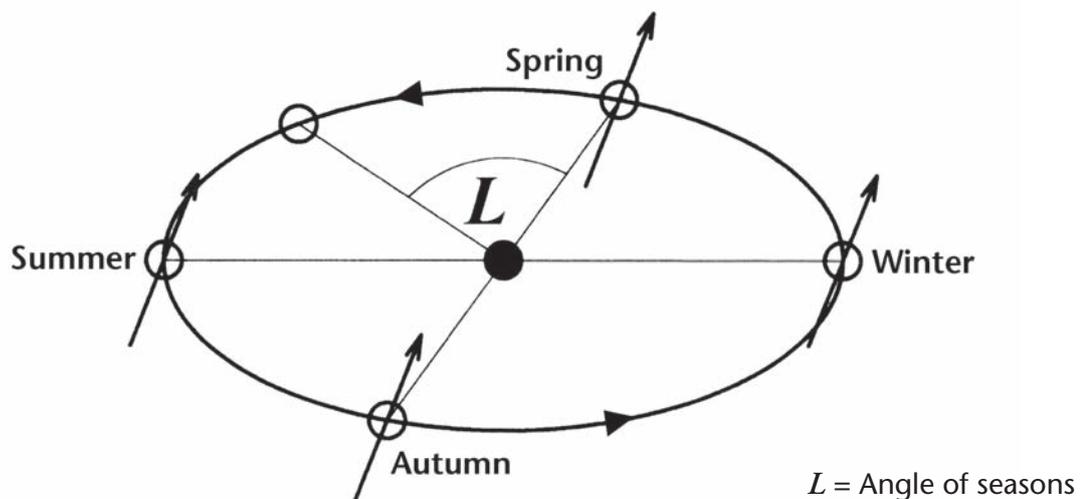
7. The lengths of day and night at various latitudes

(Teaching unit by M. Deutschendorf)

The starting position of the Tellurium for considering the lengths of the day is north summer. The point of the red month-indicator points to June 21.

We roll out 3 blue and red **bulges of modelling-clay (Plasticine)** (about 0,5 up to 1 cm thick) of a length of about 30 cm.

The pupils put those Plasticine bulges on the light/shadow border line along the latitudes.

We need to take particular care that the entire bulge lies over the respective latitude.

A **red bulge** is put along the latitude of the sunlit side, the day-side. A **blue bulge** is laid along the latitude of the unlit side, the night-side. The pupils cut the length at the respective place.

The same is done for each further quarter of the year, i.e., the point of the red month-indicator below the globe is next put on September 23, then on December 21, and finally on March 21.
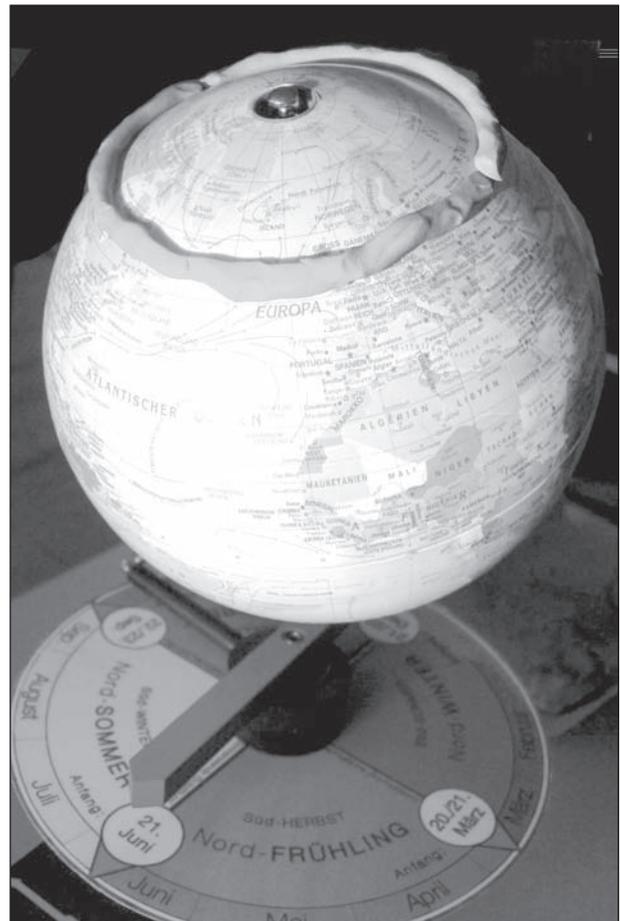
Then the pupils stick the bulges to the blackboard.

On the blackboard is seen:

In the north summer (June 21)

	day (red)	night (blue)	length of day/night
Equator	——————	——————	both are of equal length
Central Europe	————	———	longer/shorter
Arctic (at 80°)	⬭ *		24 hours/no night

In the north autumn (September 23)

	day (red)	night (blue)	length of day/night
Equator	————	————	both are of equal length (12 hours)
Central Europe	———	———	both are of equal length (12 hours)
Arctic (at 80°)	——	——	both are of equal lelngth (12 hours)

* The bulge is stuck as a circle to the blackboard so as to demonstrate thalt the sun shines all day

In the north winter (December 21)

	day (red)	night (blue)	length of day/night
Equator	————	————	both are of equal length (12 hours)
Central Europe	———	———	shorter/longer
Arctic (at 80°)		⬭	no day/24 hours

In the north spring (March 21)

	day (red)	night (blue)	length of day/night
Equator	—————	—————	both are of equal length (12 hours)
Central Europe	————	————	both are of equal length (12 hours)
Arctic (at 80°)	———	———	both are of equal length (12 hours)

Later on the teacher replaces the bulges by a coloured chalk line. The class-mates start making entries into their copy-books whereas in the front the bulges are being stuck to the blackboard.

Result: At the equator, day and night are of equal length all through the year (cp. teaching unit 2.5). In the medium latitudes the lengths of day and night differ considerably from each other in summer and winter. At the pole there is no change at all from day to night.

At the time of the equinoxes in Sept. and March we have 12 hours of day and 12 hours of night all over the world.

On page 20 you will find the work-sheet "lengths of the day at various seasons" as a copy pattern.

You can order coloured Plasticine with Cornelsen Experimenta under the item number 70200.

Further astronomic reflections

We already considered the length of day and night as a function of the seasons and the latitude in section 2.

The sun has a diameter of about 1.392 million km. At a mean distance from earth of 149.6 million km, the sun appears to us as a disc of angular diameter of 0.53° or 32' (mean value). We see the sun already when the center of the sun is still ¼° below the horizon. In addition, the refraction of the light in the atmosphere of the earth near the horizon by about ½° results in seeing the sun even a bit earlier. We notice the light rays of the sun already, when the sun's center is still ¾° below the horizon.

Without these two facts, day and night at the spring and autumn equinox would be exactly equal: 12 h in both cases. **Equinox means equality of day and night**. Taking both effects into account, the day in central Europe on March 21 or September 23 is about 24 min longer than the night. The time when really day and night have equal length is about 3 days before the spring equinox and 3 days after the autumn equinox. At the equator, day and night are 12 h long all year round, not taking into account the refraction of the light and the finite diameter of the sun. Including these, the day at the equator is about 15 min longer than the night, all year long.

Copy pattern – work-sheet: Lengths of the day at various seasons

In the north summer (June 21)

	day (red)	night (blue)	length of day/night
Equator			
Central Europe			
Arctic (at 80°)			

In the north autumn (September 23)

	day (red)	night (blue)	length of day/night
Equator			
Central Europe			
Arctic (at 80°)			

In the north winter (December 21)

	day (red)	night (blue)	length of day/night
Equator			
Central Europe			
Arctic (at 80°)			

In the north spring (March 21)

	day (red)	night (blue)	length of day/night
Equator			
Central Europe			
Arctic (at 80°)			

8. The Time of day

We put the Tellurium on **north spring** (March 21)

1. Medium Latitudes

We turn the globe in such a way that Europe points to the sun. We choose the 15th meridian for it and put the shadow-figure with the horizon-disc on **Berlin**. Now we turn the globe to the western **light/shadow border line**, i.e., entirely to the left.

We take care that the north mark of the horizon-disc points to the North Pole.

The at first very long shadow falls exactly to the west in this season. We turn the globe anti-clockwise into the sun. The shadow gradually wanders northwards , shortening all the while until at noon the shadow of the head of the figure appears yet on the horizon-disc. So, in this season the shadow is a bit longer than the figure, to be precise, it goes a little beyond 1:1.

At noon with us the shadow falls to the north in the direction of the geographic North Pole because the sun shines from the south.

If we keep turning into the time of the afternoon, the shadow gradually changes via north-east towards the east becoming longer and longer.
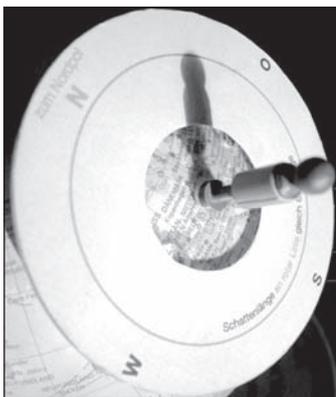
He who wants to be precise about it, may count the hours which Berlin is distant in the afternoon from the moment of midday. While doing so, without turning the globe, we put the shadow-figure on the place at the equator where there is no shadow.. At that place the meridian has noon. By counting the lines of meridian (every 15 degrees , cp. teaching unit 3.3) as far as Berlin, we find out the difference of hours which is to be added to the time of noon in order to obtain the time of the afternoon.

2. Tropic

We repeat the same at **the Tropic of Cancer**. We notice that the shadow is only half as long as the figure, a sign that the sun here is clearly higher in the sky.

3. Equator

We put the horizon-disc on the western **light-shadow border line**, i.e., all the way to the left on the equator.

We take care that the north mark of the horizon-disc points to theNorth Pole.

Now we slowly turn the shadow-figure on the globe into the sun. The shadow points to the west, i.e., the sun shines from the east.

We notice that the shadow keeps getting shorter and shorter but always keeps the same direction to the west until it completely disappears at noon. The sun is now vertical above the equator (at its zenith).

Going on turning the globe a shadow is created directing to the east. The shadow becomes longer still pointing to the east.

We register that the sun seems to run straight over the sky. It arises in the morning in the east, climbs vertically in the sky, continues its way in the afternoon and sets in the west with a long shadow. In the morning its angle against the north direction (azimuth) is 90° to increase after noon to 270°. The balance is 180°. The path keeps straight.

We can repeat this exercise for all the seasons. The results will be different concerning the shadow lengths and directions.

E.g. we adjust the tellurium arm to the **south summer** (December 21) and set the horizon disc on Cape Town / South Africa. We observe that the sun describes a small curve to the north. The short shadow at noon directs to the south (compare unit 3.2).

Further astronomic reflections

We give a few comments here concerning the apparent orbits of the sun and the stars on the celestial sphere.

As a result of earth's rotation, the stars appear to more daily from East to West. Only the northern and the southern celestial poles remain fixed. Let us consider an observer at northern latitude φ = 50°N (e.g. in central Europe). The angle of the northern celestial pole with the northerly direction of the horizon, the **polar hight**, is the same as the geographical latitude φ. All stars having angular distance less than φ from the celestial pole, seem to move in small circles around the pole which stay above the observer's horizon all day long. Stars in angular distance larger than φ from the northern pole (but less than 180° – φ), seem to move in circular arcs, rising at the eastern horizon, culminating in the south and setting in the western horizon. All apparent orbits of the stars happen in planes which form an angle of 90° – φ with the horizon. At the equator (φ = 0°) the orbital plane of the stars is perpendicular to the horizon, at the north pole (φ = 90°) parallel to the horizon – stars viewable at the north pole stay above the horizon all day long.

Due to the immense distance of the fixed stars from the earth the tilt of earth's axis relative to the horizon of fixed stars does **not** change (in any noticeable way) with the seasons. This, however, is not true for the sun – our star – which earth orbits once per year. As a consequence of this, the daily apparent orbital plane of the sun moves from north-east – north-west at the beginning of summer to south-east – south-west at the beginning of winter. The circular arc of the sun during winter is, of course, much smaller than during the summer, the period of daylight being correspondingly shorter.
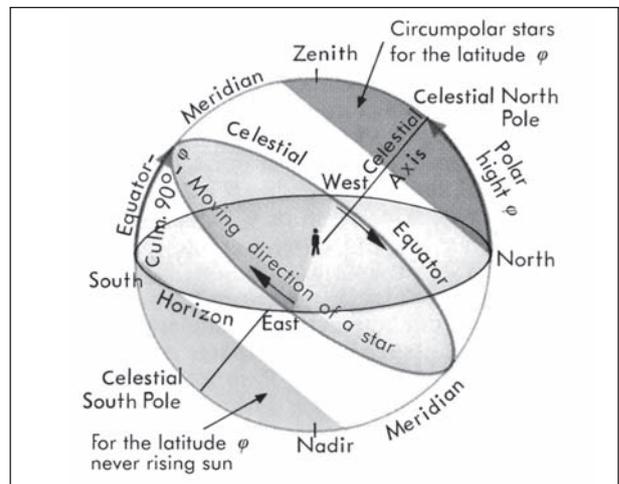
Apparent orbital plane of the sun to an observer based at latitude 50°, 66.5°, 90°, 23.5°

Apparent orbits of the stars to an observer based at latitude 50°, 66.5°, 90°, 23.5°

Illustrations on this page taken from J. Herrmann: *dtv-Atlas Astronomie;* modified
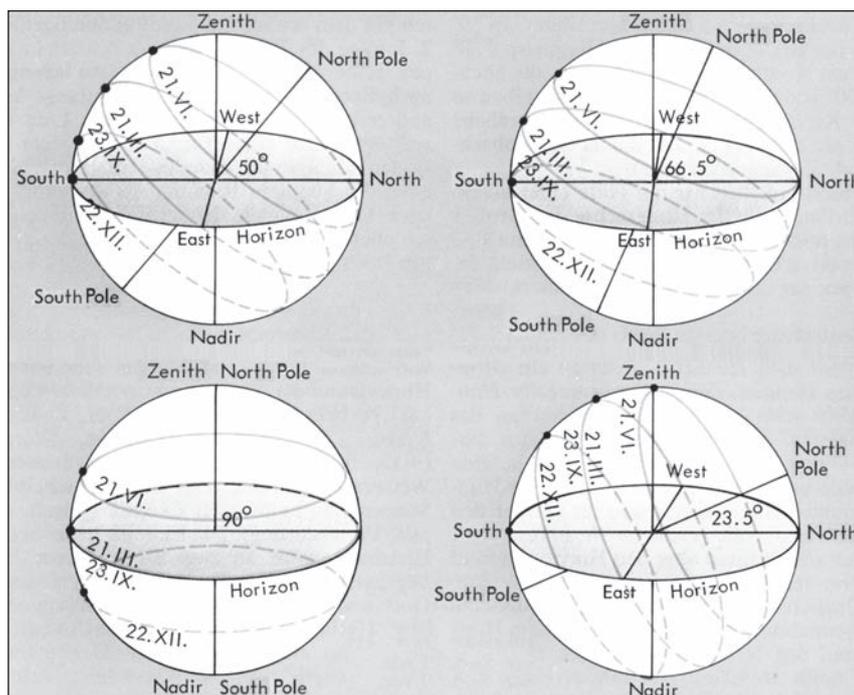
9. The phases of the moon

For this teaching unit the Fresnel-lens is to be put on position 2 „moon".

1. The moon always turns only one side to us

Apart from some slight oscillation at the edges, the moon always turns only one side to us. We can very well see it on the Tellurium.

For technical reasons the moon is not led around the earth in the right inclination. That, however, can be simulated by pulling out respectively pushing in the telescope rod.

We turn the moon to the side opposite the sun and pull out the holding rod a little so as to get the sun-light shine fully on the moon. With the enclosed washable felt pen we draw a cross into the centre of the lit side. We push the moon-rod in again and make the moon slowly carry out an entire rotation around the earth (anti-clockwise). We can see that the cross is always turned towards the earth, whether this side is lit or not.

Result: The moon always shows us the same side.

Tip: Only in 1959 were the first photos of the back of the moon made by the Russians. Therefore on the back of the moon there are names like „Lomonossow-mountains" or „Moscow-sea".

2. The phases of the moon

The pupils describe how we see the moon from the earth. For that purpose the shadow-figure on the globe is to be put opposite the moon.

a) We begin with the **new moon**.

The moon is between the earth and the sun.

We cannot see the moon because it turns to us its dark side.

b) Almost a week later we have **crescent moon**.

We carry out a quarter turn with the help of the moon-holding rod (anti-clockwise).

Seen from the earth, only one half of the moon -"disc" is lit. There is a half moon.

c) Then there is the **full moon**.

For that purpose the moon-rod needs to be pulled out, otherwise there is a lunar eclipse.
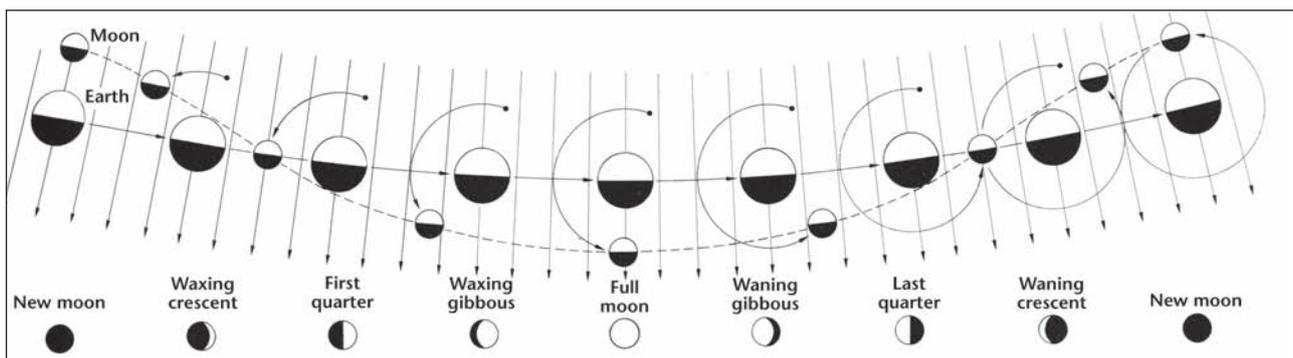
From the earth the moon is visible as a full circular disc.

d) A week later there is a **waning moon**. The lit side has the shape of a small „a". It helps us to remember which phase of the half moon there is.
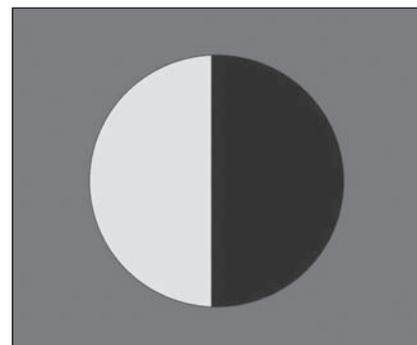
w"a"ning moon

Phases of the moon (taken from J. und S. Mitton, *Astronomie*, Christian Verlag; modified)

In the past the moon was an important month-indicator, for it was easily visible to all people. The full moon served as evening-lighting and that is why lots of feasts were fixed for the time of full moon. Numerous religions still today know holidays which follow the phases of the moon, above all in the Islam. Thus Ramadan, the month of fasting, wanders, in the course of the year, through all the year because it is not adapted to the solar calendar.

Our neighbouring planets, too, from our point of view are at variable times differently fully lit by the sun and so change their brightness.

Further astronomic reflections

The orbit of the moon is a result essentially of the gravitation of sun, earth and the moon. The lunar diameter is 3,476 km, slightly more than one quarter of the earth's diameter of 12,742 km; the moon has only 1/81 th of the mass of the earth. Relative to the earth, the moon moves in roughly elliptical orbits around their center of gravity which is still inside the earth at a distance of about 4,500 km from the center of the earth. Superimposed on this movement clearly is the orbit of the center of gravity of earth and moon around the sun. The plane of the lunar orbit is inclined by 5.15° against the ecliptic, the plane of the orbit of the earth around the sun. Hence the lunar orbital plane is inclined against the equatorial plane between 18.3° (= 23.45° − 5,15°) and 28.6° (= 23.45° + 5.15°), varying in time. The excentricity of the lunar orbit around earth is higher than the one of earth's orbit around the sun: whereas the distance earth-sun only varies by at most 3.3 % over a year, the distance moon-earth varies by as much as 14 % between 356,400 km and 406,700 km, with a mean value of 384,400 km.

Since the moon does not emit light, but only reflects sunlight, it is invisible for us on earth while being in the new moon position between sun and earth. In the full moon position, viewed from earth opposite to the new moon position, it is illuminated fully by the sun: in general the moon is substantially north or south of the ecliptic (inclination of the orbit 5.15°). If the moon is very close to the ecliptic in the full moon position, it may pass the shadow cone of the earth and a lunar eclipse results. In the half moon position, halfway between new and full moon, the intensity of the reflected light is only 1/9 th of the one of the full moon; however, the contrast, in particular, close to the lunar rim, is much higher than at full moon.

In winter the sun in central Europe (e.g. Berlin φ = 52.5° N) rises only about 14° above the horizon. At full moon in winter, however, the earth's axis is tilted towards the moon, being on the opposite side of the sun. Thus the moon is very high above the horizon then, up to 61° ± 5° (61° = 90° − 52.5° + 23.5°). In summer, a full moon rises only very little above the horizon (14° ± 5°).

A **lunation** is the mean time between one full moon and the next full moon. This "full-moon-month" is 29.5306 days = 29 d 12 h 44 min long. It is an average value; in a particular case the period of time between two successive full moons my be up to 7 hours longer or shorter. The **sideral month** is the mean period of time, after which the moon attains the same position (longitude) with respect to the fixed stars. The sideral month lasts 27.3216 days = 27 d 7 h and 43 min. It is hence more than 2 days shorter than the lunation: after 27.3 days the moon in its orbit reached the same longitude relative to the fixed stars again, but the longitude of the sun relative to the earth changed by about 27° due to earth's orbit around the sun. Up to the next full moon position the moon has to pass another angular arc.

The period of rotation of the moon around its axis coincides with the period of rotation of the moon around the earth. **This "fixed" rotation implies that we always see the same side of the moon on earth.** We cannot observe (most of) the back side of the moon. Some areas close to the rim of the moon between the front and the back side may be viewed at certain times, however, due to the **libration**. Altogether 59 % of the area of the moon can be seen from earth at least during certain times. The libration consists of two different phenomena: the libration in longitude yields a rotation of the moon relative to the earth by up to ±8°, since the moon rotates evenly around its axis, but not evenly around the earth (the latter depending on the particular distance to the earth). The libration in latitude is a result of the inclination of the lunar equator against the lunar orbital plane by about 6 ⅔°. Therefore sometimes the north pole of the moon, sometimes the south pole of the moon are more tilted towards the earth.

Siderial month (AX) and lunation (AB)
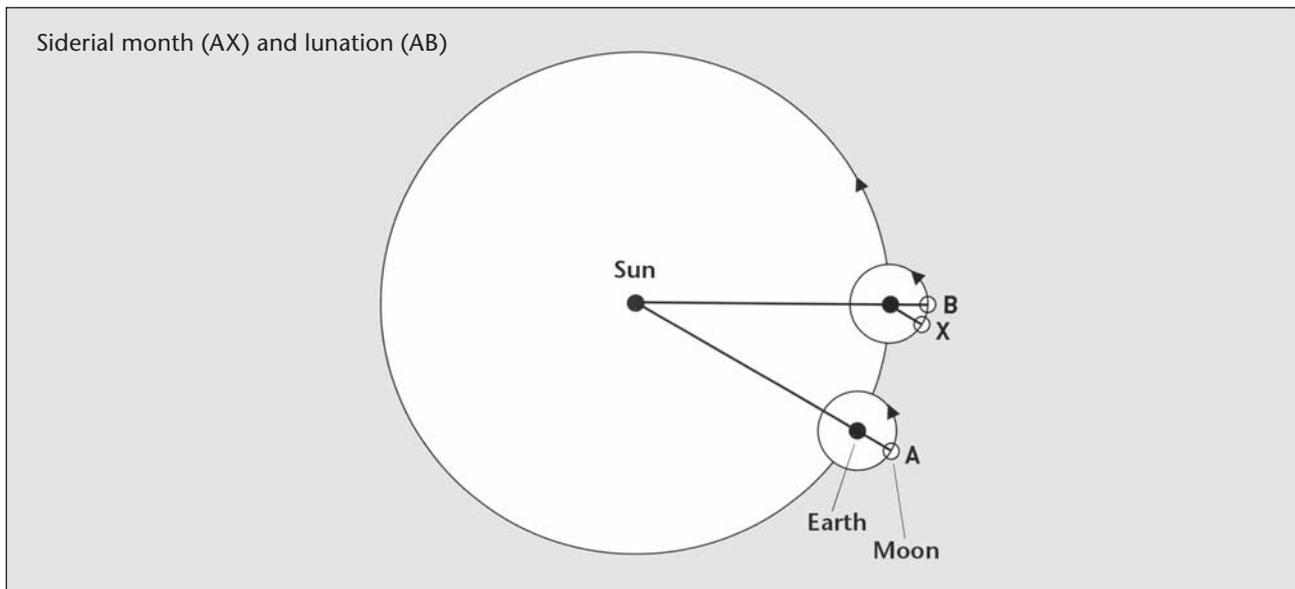
10. The eclipses

For this teaching unit the Fresnel-lens is to be put on position 2 „moon".

1. Lunar eclipse

When sun, earth, and moon, in this order, are exactly in one line, the shadow of the earth covers the moon. Instead of full moon we have a **lunar eclipse**.

> **Tip:** The moon-rod remains inserted. Because of the short distance between the earth and the moon on the Tellurium, by mistake every month instead of a full moon develops a lunar eclipse. As in reality the distance is much longer (it ought to be here about 4.5 m) and because of the fact that the moon describes an inclined path around the earth, in nature lunar eclipses happen only very seldom.
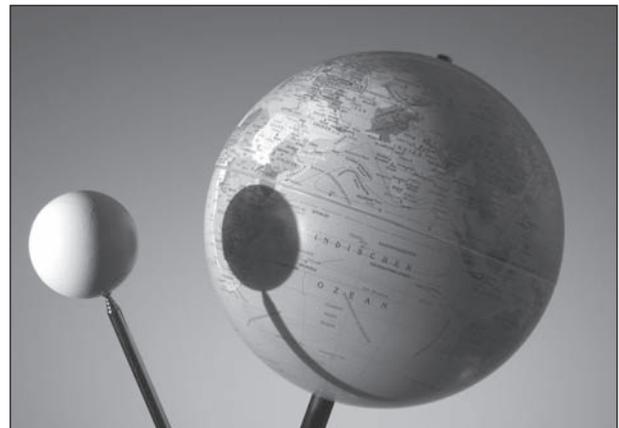

2. Solar eclipse

We put the moon exactly between the earth and the sun. Its shadow falls on the earth and darkens some part of it. We have a **solar eclipse** for in the area of the shadow we cannot see the sun.

We put the horizon-disc with the shadow-figure between the earth and the moon. It, as well as its surroundings, remains unlit. For many minutes, in broad daylight, there is a condition similar to night. (Turn the globe) In former times that unusual phenomenon was regarded as a harbinger of great, mostly negatively set events.

That condition, too, happens only seldom in nature.

Further astronomic reflections

During a total solar eclipse the moon covers the sun completely for observers on a narrow long band on earth and the solar **corona** becomes visible. During a lunar eclipse the moon enters the shadow cone of the earth. Both eclipses are possible since moon and sun appear to the observer on earth as discs of almost the same angular diameter of about $\theta = \frac{1}{2}°$. The sun has a diameter 400 times as large as the moon; by coincidence, it is on average also about 400 times as far away from earth as the moon. The solar radius is $R = 696,000$ km, the lunar radius $r = 1,738$ km. The mean solar distance is 149.6 million km, the mean lunar distance is 384,400 km. Since the distance earth–sun varies by 3.3 % over the year, and the distance moon - earth varies by up to 14 %, the apparent angular diameters of the sun θ_S and the moon θ_M for observers on the surface of the earth vary between
$0.524° \leq \theta_S \leq 0.542°$ and $0.497° \leq \theta_M \leq 0.567°$,
with mean values of $\theta_S = 0.533°$ and $\theta_M = 0.527°$. On average, the moon appears to the observer on earth slightly smaller than the sun.

Eclipses are possible only, if sun, moon and earth are in one line: solar eclipses, if the moon moves between sun and earth, lunar eclipses, if the earth enters between sun and moon.

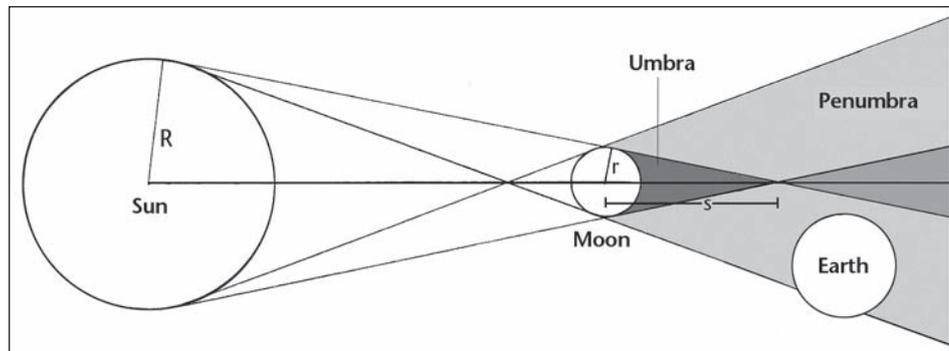## Umbra and penumbra: total, annular and partial solar eclipses

Total solar eclipse:
Moon relatively near to
the earth, umbra meets
the earth fully

Diameter of the umbra q up to 260 km
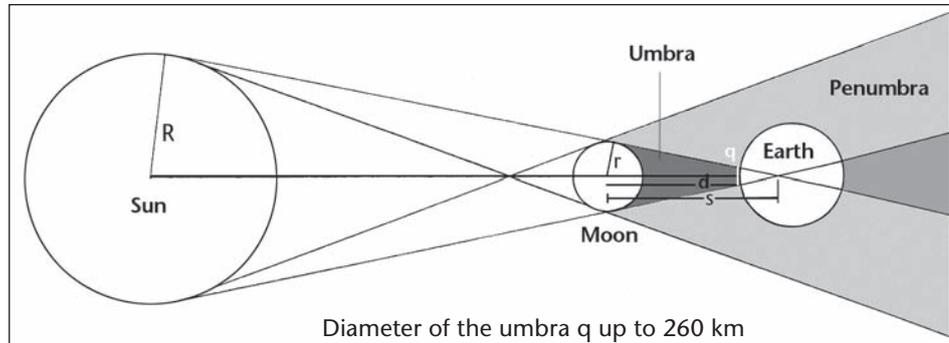
Annular solar eclipse:
Moon relatively far
from earth

Diameter of the umbra q up to 320 km
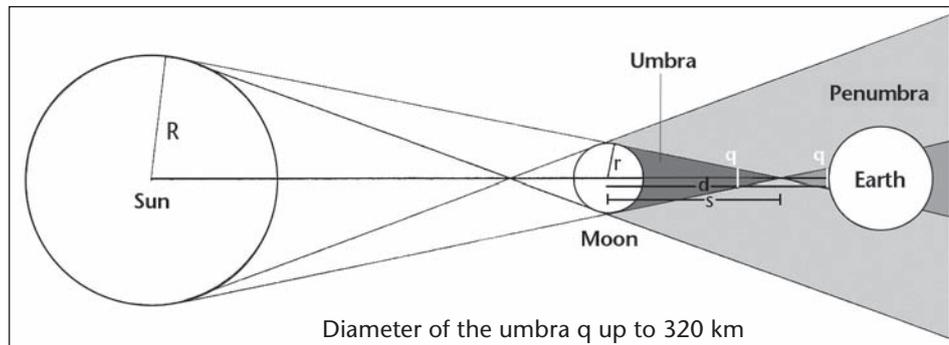
Partial solar eclipse:
Moon relatively high,
umbra doesn't meet
the earth

Solar eclipses thus only happen during the new moon phase, lunar eclipses during the full moon phase. Sine the lunar orbit is inclined by 5.15° against the ecliptic, the moon usually is far north or south of the ecliptic during new or full moon, then no eclipse occurs. Only in case that the moon happens to be close to the **nodal line** (the line of intersection of the lunar orbital plane and the ecliptic), it is close enough to the ecliptic to cause an eclipse.

If the moon then is positioned between sun and earth and the lunar angle θ_M is larger than the solar angle θ_S, one observes a **total solar eclipse** in a narrow zone on earth. In the same configuration but with angle θ_M smaller than

θ_S, an **annular solar eclipse** is the result: the moon does not cover the sun completely, but leaves a narrow ring of the sun visible around the moon (again in a narrow zone on earth). Annular solar eclipses are somewhat more frequent than total solar eclipses.

During a **partial solar eclipse**, the moon only partly covers the sun for an observer on earth – the observer, moon and sun are not exactly on one line but almost so. The nodal line is not fixed in space but moves slowly clockwise against the fixed star horizon – once in 18.6 years. Thus the earth during its orbit around the sun passes the zone of the nodal line twice per year (more precisely twice every 346.6 days):

in this period eclipses can occur. In any year, there will be at least two partial or central (= total or annular) solar eclipses and at most five such eclipses: at most two when passing a "danger zone" close to the nodal line and one more during the remaining 18.6 days (365.2 – 346.6) of the calendar year. The number of lunar eclipses per year – partial or total – is between 0 and 3.

The vertex of the moon's shadow cone has distance s from the moon, where 367,300 km $\leq s \leq$ 379,800 km holds. If the moon during the new moon phase is close enough to the nodal line and has a distance d < s from earth, a total solar eclipse results. The **umbra** (shadow cone of the moon) has a diameter ρ determined by

$$\rho = 2\,r - \frac{2\,d\,(R - r)}{(D - d)}$$

d = lunar distance, D = solar distance

This value is between 0 and 260 km (provided d < s): the umbra meets earth in the band of totality. If d > s, ρ < 0 and in the rear part of the umbra of diameter $|\rho|$, 0 < $|\rho|$ < 320 km

one may observe an annular eclipse. Total solar eclipses may last near the equator up to 7 ½ min, in central Europe ($\varphi \approx$ 50° N) only up to 5 ¾ min. The reason for the difference is the higher rotational speed of the earth at the equator which diminishes the relative speed of the umbra over the earth. Typically in central Europe, total eclipses only last 2–3 minutes.

Since the earth is much larger than the moon, the shadow cone of the earth is much longer than the one of the moon, namely about 1.4 million km. In its typical distance from earth (384,000 km), the moon easily fits into the shadow cone of the earth (in case of a full moon phase close to the nodal line). The resulting lunar eclipse may last up to 1 ¾ h. A total lunar eclipse may be watched from the night side of the earth whereas central solar eclipses only occur on a small band on earth (typically 100–150 km wide and 12,000 km long).

The next total solar eclipse in central Europe is on September 3, 2081 (France, Southern Germany, Switzerland, Austria), then on October 7, 2135 (Northern Germany).
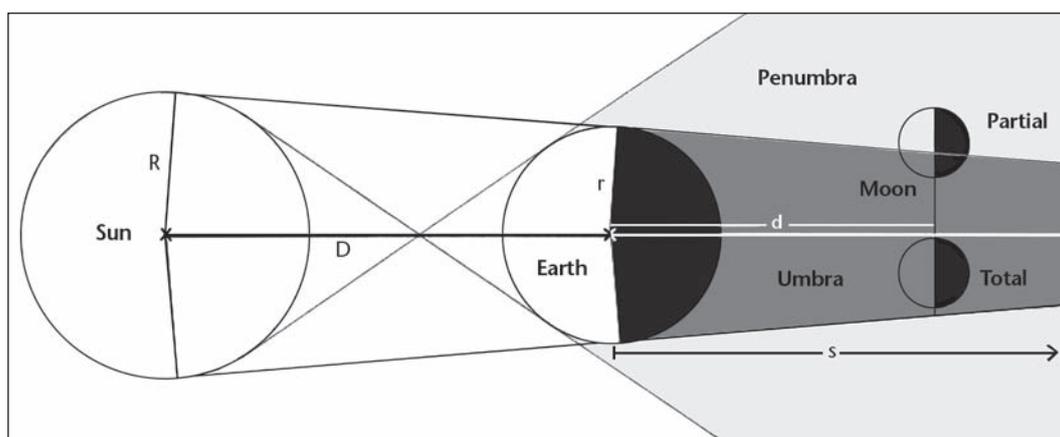
Shadow cone and lunar eclipses

11. The tides

When sun, moon, and earth are in one line, then because of the added tidal energy, there are particularly high daily high tides in the oceans and seas.

We call this event **spring tide**. As such a lining up of the heavenly bodies happens at full moon as well as at new moon, about every fortnight there is a spring tide (s. picture). We can illustrate that phenomenon as follows:

1. We put the moon in full-moon position. It is then in a line with the earth and the sun.

2. With the washable felt pen we draw a line from pole to pole via Greenwich near London along the prime meridian and lengthen it on the opposite side to a full circle (180th meridian). We do the same at the 90th east and west meridians and thus have divided the earth into 4 equal parts.

3. We turn the prime meridian towards the sun.

4. We put a somewhat wider tape or string, which is 2 inches longer than the globe, around the pole and have it tightened by 2 pupils, i.e., drawn towards the sun respectively off the sun. At the primemeridian and at the 180th meridian we now have spring tide.

A third pupil now turns the earth underneath the tape (or string).

After a quarter turn, i.e., after 6 hours, the height of the tide is at America and Asia.

After another quarter turn, i.e., after twice 6 hours =12 hours, we have spring tide again along the starting line.

So in 24 hours the earth turns about twice underneath the high-water „mountains" respectively the low-water „valleys".

Therefore on the oceans there are twice daily the highest tidal levels (daily high tide) and the low-water levels (daily low tides).

> **Tip:** The causes in detail are very much more complex, i.e., it is not only a question of forces of gravitation respectively attraction but also of centrifugal forces. In this process earth and moon form a system of gravitation whose common centre of gravity, because of the mass of the earth, lies below the surface of the earth. As the moon moves on a bit every day, two tides do not last exactly 24 hours but about one hour longer.

A week after the spring tide earth, sun, and moon are at right angles to each other. We have half moon. The forces generating the tides get weaker. Now all over the world is **neap tide**. At neap tide the high-water levels are lower and the low-water levels a little higher, i.e., the difference between high and low water, the tidal range, is smaller.

> **Tip:** The moon-conditioned forces are stronger than the sun-conditioned ones.

The differences between spring tide and neap tide can be some decimetres on the German North Sea coast. The concurrence with an accidentally strong storm-event leads to particularly high water levels on the coast. Such **spring tides** are especially dreaded.
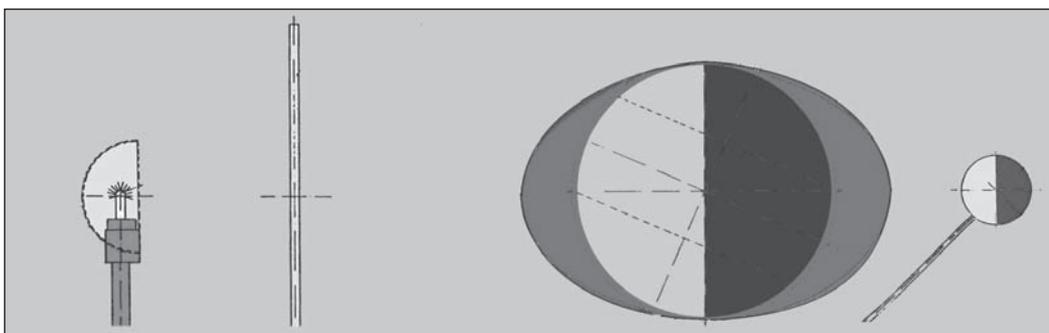

Spring tide with much exaggerated wave "mountains" (Crests of waves).

Further astronomic reflections

Ebb and flow tide are caused by the gravitational forces of sun, earth and moon. The effect of the moon–earth gravitation on the ocean water dominates the sun–earth effect, since the moon is much closer to the earth than the sun. Principally, not only the ocean water is attracted by the moon, but the continents are, too, but with a much smaller amplitude. The essential periodicity of the moon dominates the frequency of the tides: the mean time between two passings of the meridian by the moon amounts to 24 h and 50 min: the earth turns in 24 h once around its axis, but moon moved by about 13° in this time on its orbit around the earth; hence the earth has to turn for another 50 min until the next meridian passage of the moon. Since the speed of the moon in its orbit varies considerably (the mean excentricity of its orbit is 0.0549), the 24 h and 50 min constitute only a mean value for a "lunar day". Thus the time of 12 h and 25 min is only a mean value for both tidal periods per lunar day.

The tidal range and exact time of high and low tide naturally also depend heavily on the coastal geography. Close to the German North Sea coastline the tidal range is around 3 m during spring tides and slightly below 2 m during neap tides (mean values). The friction of the mass of water hitting the coastal shores results in a loss of the rotational energy and of the angular momentum of the earth: the speed of the rotation of the earth very slowly diminishes over millions of years, and the mean solar day increases by about 1.7 m sec per century (by the tidal influence it should even increase by 2.3 m sec per century, this is partially compensated by effects still being under investigation, e.g. the uplift of masses of land after the last ice age). Since our "day" was defined in 1900, currently on average every 600 days ≈ 1.6 years (1000 ÷ 1.7) a leap second has to be inserted: the clock is stopped for a second either after June 30 or December 31.

Since the angular momentum of the system sun–earth–moon is constant and since this also applies to the subsystems sun–earth and earth–moon, the decreasing angular momentum of the earth's rotation is compensated by an increasing angular momentum of the moon's orbit around the earth. This implies that the moon very slowly recedes from the earth; on average the mean distance earth–moon increases by 3.7 m per century.
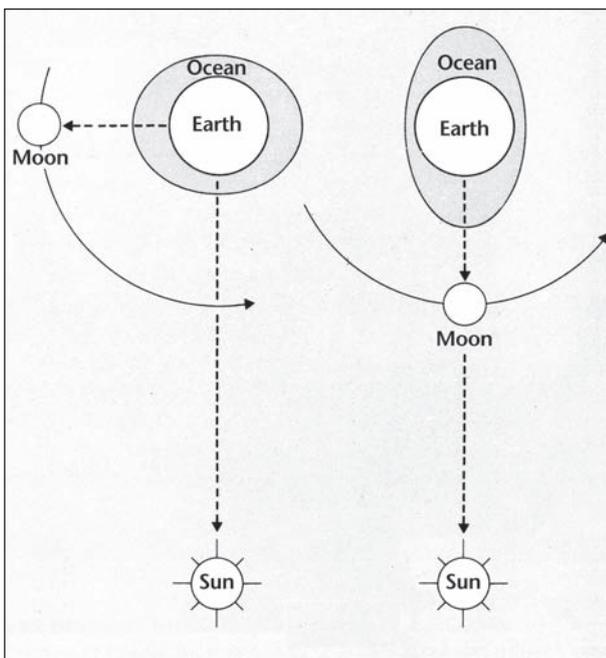

Moon, sun and intertidal effects
(taken from J. und S. Mitton: *Astronomie*, Christian Verlag; modified)

12. Measuring the earth´s circumference

The size of the earth cannot be determined by direct measuring, so some indirect measuring is necessary. The Greek astronomer Eratosthenes measured the earth's diameter in 250 B.C. with the help of a simple geometric law. He knew that on the day of the summer solstice at noon the sun was at its zenith above the town of Assuan (A)(no casting of shadow), for there was a fountain near Syene (today Assuan) in which, once a year, the sun was reflected. On the same day, at the same time, the sun in Alexandria (B) cast a shadow which indicated that the sun's position deviated by 7° degrees from the zenith.

Eratosthenes concluded that rays of light which come from a distant source of light hit the earth parallel, and as he knew that the distance between the two towns was about 800 km (he measured 5,000 stadiums), he was able to calculate the size of the earth as follows:

7° degrees is about 1/50′ of the total earth's circumference (360 degrees). The earth's circumference accordingly must be 50 x 800 km (approximate distance between Assuan and Alexandria) = about 40,000 km.
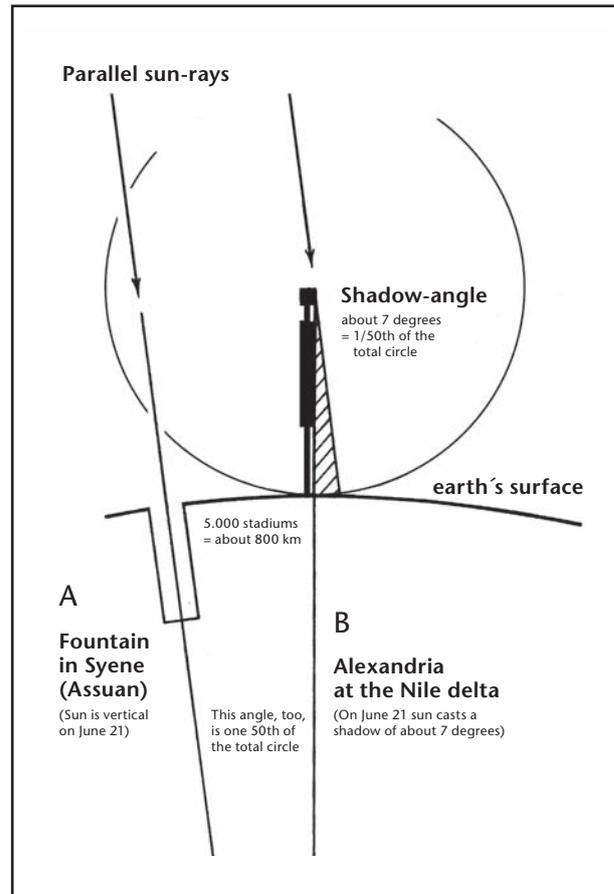
We put the Tellurium on June 21 (north summer) and the shadow-figure on Assuan. We turn the globe to and fro till the figure casts no longer any shadow. Now the sun is vertical over Assuan and we have midday in Assuan.

Now we put the figure on Alexandria. It casts a short shadow to the north.

We measure the distance from Assuan to Alexandria with the help of a piece of graph paper. The distance is 9 to 10 mm. Given a globe scale of 1: 85 million, this is 800 km.

The shadow-figure is 29 mm tall and we regard it as the radius of a circle with a circumference of 182 mm. One degree corresponds to 182:360, i.e., roughly 0.5 mm.

We put the graph paper at the foot of the shadow-figure and read a shadow length of between 3 and 4 mm. With 3.5 mm we get an angle of 7° degrees. That is the angle which also Erastosthenes calculated.

Tip: Connected with the Enlightenment in France they created the metre as artificial unit of measurement. They equalled a quarter of the earth's circumference, i.e., once the distance from the equator to the pole, with 10,000 km. They again divided 1 km in one thousand parts and thus got the metre as the 10 millionth part of the earth quadrant.

Further astronomic reflections

It was known already in antiquity that the earth has the shape of a ball. Signs of evidence for this were firstly the circular shadow of the earth on the moon during lunar eclipses, and secondly, that one could see only higher parts of coastal lands while approaching a coastline by ship from the sea.

Eratosthenes was one of the most versatile scholars of antiquity; he lived in Alexandria in the third century BC. Eratosthenes knew that at the summer solstice the sun stood vertically above Syene (Assuan) at midday. Assuan is next to the Tropic of Cancer, Alexandria has about the same latitude as Assuan but is situated further north. Thus the sun passed the meridian at about the same time. Eratosthenes measured the shadow of a gnomon (rod) and this way determined that the direction of the sun formed an angle of 7.2° against the zenith midday at the summer solstice in Alexandria. The distance Alexandria – Syene therefore had to be $1/50$ (= 360° ÷ 7.2°) of the circumference of the earth. The distance Alexandria – Syene was 5,000 stadiums, resulting in 250,000 stadiums for the circumference of the earth. The exact value of a stadium in meters is not known very precisely, probably 250,000 stadiums are between 37,000 km and 37,500 km. Eratosthenes' value of earth's circumference thus may have been to small by a small margin, due to an imprecise value of the distance Alexandria – Syene. Further, the latitude of Syene (24.1° N) did not coincide exactly with the obliquity of the ecliptic of that time (23.8° N).

13. Geo-stationary satellite

Preparations:

The red month-indicator is put on December and the moon, so as not to be interfering, on the month of May. The Fresnel lens is taken off.

The rod with ball-end is stuck on the figure of the horizon-disc and the horizon-disc with the rod on the equator south of Europe is put on the globe.

We imagine the ball at the end of the rod to be an earth satellite. With a distance of exactly 42.2 cm from the earth it is, compared with a real satellite, distant true to scale. With a diameter of the globe of 15 cm and a corresponding circumference of the globe of 47.1 cm the distance of the satellite from the earth's surface is a little less than the earth's circumference.

The true sizes are:
Circumference of the earth: 40,000 km

Distance between the satellite
and the earth's surface: 36,000 km

How does the distance of 36,000 km come about? The satellite is acted on by the force of the earth's gravitational pull and the centrifugal force. The force of the earth's gravitational pull would like to pull it down on the earth, the centrifugal force to centrifuge it into space. At a distance of 36,000 km the force of the earth's gravitational pull and the centrifugal force of the satellite are more or less equal. The satellite is arrested between those two forces and orbits the earth always in the same position above the equator. As only above the equator both forces always have the same effect, geo-stationary satellites can maintain their position only here and nowhere else.

We slowly turn the globe with the satellite anti-clockwise. The Tellurium shows conclusively that the earth-satellite always keeps its position above the equator. We call that position geo-stationary. Via such a satellite we can continuously broad-cast on a certain invariable reception area. There must always be visual contact to the satellite, i.e., houses or trees in the visual beam interfere with the reception.

In 24 hours the earth turns once around itself during which time the horizon-disc-figure on the equator covers 40,000 km. Thus it has a speed of 40,000 : 24 = 1,667 km/h. As for the satellite we have to consider the 36,000 km above the earth's surface as well as 6,300 km as far as the earth's centre. That is a radius of 42,300 km. The circumference is (C=2 π r) about 265,600 km. Divided on 24 hours it makes about 11,000 km.

So the satellite rotates through space at a speed of 11,000 km/h.

If we put a somewhat longer (50 cm) ruler against it, we can see that at the poles we can no longer establish any visual contact. So in the regions about the poles no TV program can be received from a geo-stationary satellite.

The graph mentioned below demonstrates the whole situation.
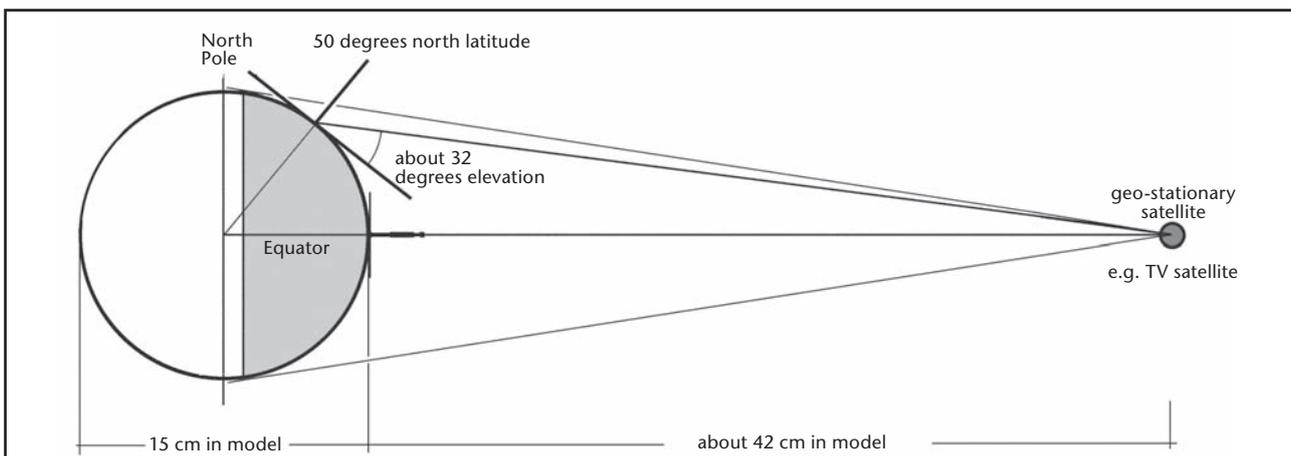
We also see that the aerials for the reception of the satellite signals with us in Germany at about 50 degrees north latitude must be inclined by about 32 degrees opposite the horizontal in order to get an optimal reception.

The satellite is always in the south and above the equator but it can possibly also have a position moved by some degrees to the west (more than 180 degrees geographical azimuth) or to the east (less than 180 degrees geographical azimuth). The Astra-satellite has a position that deviates by 19.2 degrees to the east, i.e., above the longitude of 19.2 east. Regarded from Hamburg the satellite has a position of 28.3 degrees above the horizon, in Munich 34.2 degrees, and in Frankfurt 31.7 degrees.

Results: Geo-stationary satellites always have their position above the same point above the equator at a height of 36,000 km above the earth's surface. There the centrifugal force and the earth's gravitational pull are more or less equal. In the case of more distant reception areas the signals are emitted there at an angle. In the pole areas there is no reception possible.

Further astronomic reflections

Considering the system earth-satellite as a two-body-problem, Kepler's planetary laws, Newton's laws and the law of gravitation yield a formula for the period T of rotation of a satellite around the earth

$$T = \frac{2\pi r^{3/2}}{\sqrt{\gamma M}}$$

costant of gravitation: $\gamma = \dfrac{6{,}673 \times 10^{-11}\, m^3}{kg \times s^2}$

mass of the earth: $M = 5{,}977 \times 10^{24}\, kg$

R = (larger) semiaxis of the elliptical orbit of the satellite around the earth.

To have a geostationary satellite requires T to be 24 h, T = 86,400 sec. Assuming a circular orbit of the satellite, the formula gives the distance of the satellite from the center of the earth as $R = (\gamma M)^{1/3} (T \div 2\pi)^{2/3} = 42{,}250$ km, the satellite therefore being 35,870 km above the surface of the earth (at the equator). The circular orbit of the satellite has a circumference of 265,500 km, the orbital speed is 11,060 km/h.

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